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NOTE: All diagrams are deliberately approximate. This is to ensure that the answers cannot be easily calculated by simply measuring the lengths and angles from the diagrams, and also to test the students' ability to accurately draw diagrams from given information.



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# ANGLES OF REGULAR POLYGONS

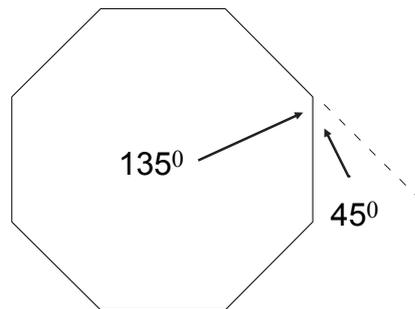
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REGULAR POLYGONS: These are shapes in which all the angles and all the sides are equal. To find angles of regular polygons you must understand the following terms:

INTERIOR ANGLES - Angles inside the regular polygon.

EXTERIOR ANGLES - Angles outside the regular polygon.

An octagon is an example of a regular polygon.



To find the exterior angles use the equation:

$$\text{Exterior angle} = 360 \div \text{number of sides}$$

$$\text{Exterior angle} = 360 \div 8$$

$$\text{Exterior angle} = 45^\circ$$

You are now able to find the interior angles of an octagon by using the equation:

$$\text{Interior angle} = 180 - \text{exterior angle}$$

$$\text{Interior angle} = 180 - 45$$

$$\text{Interior angle} = 135^\circ$$

An octagon has eight sides and therefore eight lines of symmetry. All regular polygons have the same number of lines of symmetry as the number of sides.

For example, an equilateral triangle has three sides and therefore three lines of symmetry.

## WORKED EXAMPLE

A regular polygon has an interior angle of 108 degrees, using this information find the number of sides this polygon has?

$$\text{Interior angle} = 180 - \text{Exterior angle (Rearrange equation)}$$

$$\text{Exterior angle} = 180 - \text{Interior angle}$$

$$\text{Exterior angle} = 180 - 108$$

$$\text{Exterior angle} = 72^\circ$$

$$\text{Exterior angle} = 360 \div \text{number of sides (Rearrange equation)}$$

$$\text{Number of sides} = 360 \div \text{Exterior angle}$$

$$\text{Number of sides} = 360 \div 72$$

$$\text{Number of sides} = 5$$

# ANGLES OF REGULAR POLYGONS

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# 2

## WORKED EXAMPLE

Calculate the interior angle of a hexagon, by first finding the value of the exterior angle?

Exterior angle =  $360 \div$  number of sides

Exterior angle =  $360 \div 6$

Exterior angle =  $60^\circ$

Interior angle =  $180 -$  Exterior angle

Interior angle =  $180 - 60$

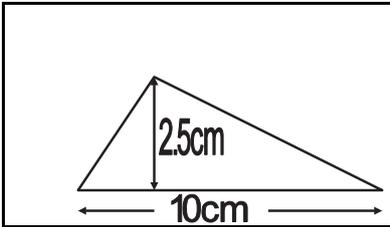
Interior angle =  $120^\circ$

## QUESTIONS

- 1) Write down the definition of a regular polygon.
- 2) A regular polygon has three lines of symmetry, name the polygon and calculate its interior angle.
- 3) Draw a square and write down how many lines of symmetry it has.
- 4) Rearrange the following equation so that the '*number of sides*' is the subject.  
Exterior angle =  $360 \div$  number of sides.
- 5) If a regular polygon has an exterior angle of 51.43 degrees to two decimal places find the number of sides that the polygon has.
- 6) If a regular polygon has an exterior angle of 40 degrees find:
  - a) The interior angle of the polygon.
  - b) The number of sides the polygon has.

# AREAS AND VOLUMES

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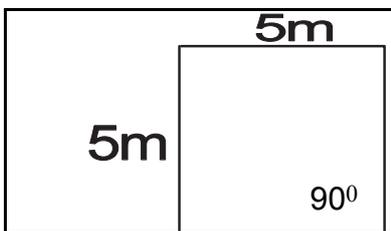


Area Of A Triangle =  $\frac{1}{2} \times \text{Base} \times \text{Height}$

WORKED EXAMPLE FOR DIAGRAM

Area Of Triangle =  $\frac{1}{2} \times 10 \times 2.5$

Area Of Triangle =  $12.5 \text{ cm}^2$



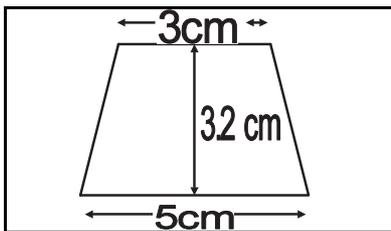
Area Of A Square =  $\text{Base} \times \text{Height}$

NB:  $\text{Base} = \text{Height}$

WORKED EXAMPLE FOR DIAGRAM

Area Of Square =  $5 \times 5$

Area Of Square =  $25 \text{ m}^2$

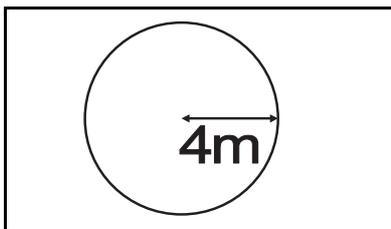


Area Of Trapezium =  $\text{Vertical Height} \times \text{Average Of Parallel sides}$

WORKED EXAMPLE FOR DIAGRAM

Area Of Trapezium =  $3.2 \times ((5 + 3) \div 2)$

Area Of Trapezium =  $12.8 \text{ cm}^2$

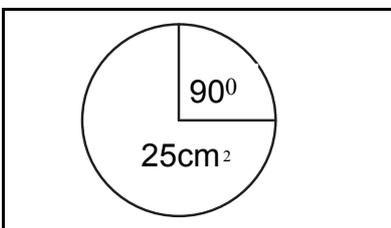


Area Of Circle =  $\pi \times \text{radius}^2$

WORKED EXAMPLE FOR DIAGRAM

Area Of Circle =  $\pi \times 4 \times 4$

Area Of Circle =  $50.27 \text{ m}^2$  (to 2 d.p.)

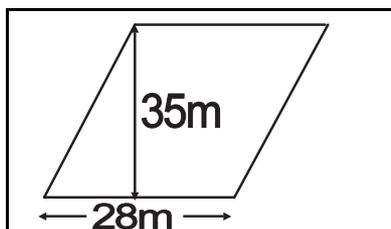


Area Of Sector =  $(\text{Angle} / 360) \times \text{Area Of Circle}$

WORKED EXAMPLE FOR DIAGRAM

Area Of Sector =  $(90 \div 360) \times 25$

Area Of Sector =  $6.25 \text{ cm}^2$



Area Of A Parallelogram =  $\text{Base} \times \text{Height}$

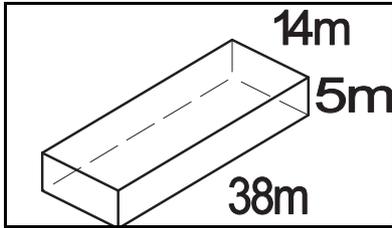
WORKED EXAMPLE FOR DIAGRAM

Area Of Parallelogram =  $28 \times 35$

Area Of Parallelogram =  $980 \text{ m}^2$

# AREAS AND VOLUMES

4

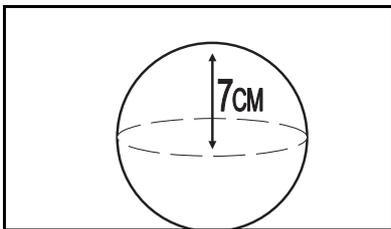


Volume Of A Prism = Area Cross Section  $\times$  length

WORKED EXAMPLE FOR DIAGRAM

$$\text{Volume Of Prism} = (14 \times 5) \times 38 = 2660 \text{ m}$$

$$\text{Volume Of Prism} = 2660\text{m}^3$$

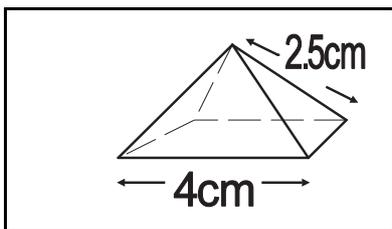


Volume Of A Sphere =  $\frac{4}{3} \times \pi \times \text{radius}^3$

WORKED EXAMPLE FOR DIAGRAM

$$\text{Volume Of Sphere} = \frac{4}{3} \times \pi \times 7^3$$

$$\text{Volume Of Sphere} = 1437\text{cm}^3 \text{ (to 4 sig.fig.)}$$

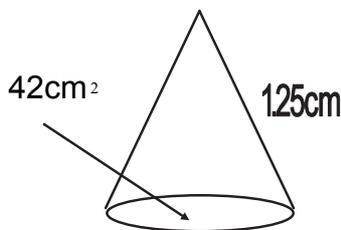


Volume Of A Pyramid =  $\frac{1}{3} \times \text{Area Of Base} \times \text{Height}$

WORKED EXAMPLE FOR DIAGRAM

$$\text{Volume Of Pyramid} = \frac{1}{3} \times 16 \times 2.5$$

$$\text{Volume Of Pyramid} = 13.33\text{cm}^3 \text{ (to 2 d.p.)}$$



Volume Of A Cone =  $\frac{1}{3} \times \text{Area Of Circular Base} \times \text{Height}$

WORKED EXAMPLE FOR DIAGRAM

$$\text{Volume Of Cone} = \frac{1}{3} \times 42 \times 1.25$$

$$\text{Volume Of Cone} = 17.5 \text{ cm}^3$$

## QUESTIONS

- 1) A triangle has a base of length 35cm and a vertical height of 45cm, find its area.
- 2) A circle has a diameter of 10cm, find the area of the circle. (NB: diameter = radius  $\times$  2)
- 3) If the area of a circle is 25cm find the area of the minor sector which has an angle of 52 degrees.
- 4) A parallelogram has a base of 15 metres and a vertical height of 12 metres: find its area.
- 5) Calculate the volume of a cylinder which has a radius of 6cm and a length of 18cm.
- 6) If a pyramid has a square base of length 2cm and a height of 12 cm; find
  - a) The area of the cross section
  - b) The volume of the pyramid
- 7) A sphere has a radius of 8.5cm. Calculate the volume of the sphere.

## Multiplying Fractions

This is very simple and only requires you to multiply the top row of numbers together followed by the bottom row of numbers.

### Worked Example

Calculate  $\frac{7}{8} \times \frac{1}{3}$

First multiply the top numbers together i.e.  $7 \times 1$ .

Next multiply the bottom numbers together i.e.  $8 \times 3$ .

So  $\frac{7}{8} \times \frac{1}{3} = \frac{7}{24}$

## Dividing Fractions

The method for dividing fractions is very similar to multiplying fractions. The only difference is that you turn the second fraction upside down first and then use the multiplication rule.

### Worked Example

Find the value of  $\frac{4}{5}$  divided by  $\frac{15}{13}$

First invert the second fraction so that the calculation reads:

$$\frac{4}{5} \times \frac{13}{15}$$

Now use the method of multiplying the numbers on the top together followed by multiplying the numbers on the bottom together.

$$4 \times 13 = 52$$

$$5 \times 15 = 75$$

$$\frac{4}{5} \times \frac{13}{15} = \frac{52}{75}$$

## Addition Of Fractions

First a common denominator must be found, which means the numbers on the bottom of the fractions must be made the same.

For example:  $\frac{5}{6} + \frac{3}{4}$

Make the bottom number the same by multiplying the 6 and the 4 together.

Then multiply the top number of the first fraction by the bottom number of the second fraction i.e.  $5 \times 4$ .

Now do the opposite; multiply the top number of the second fraction by the bottom number of the first fraction i.e.  $3 \times 6$

The equation should now read:  $(5 \times 4) \div (6 \times 4) + (3 \times 6) \div (6 \times 4)$

Now simply add the numbers on the top together as shown:

$$\frac{20}{24} + \frac{18}{24} = \frac{38}{24}$$

## Subtraction Of Fractions

This is exactly the same as addition of fractions, as once you have found the common denominator and adjusted the numbers on the top row accordingly, you subtract the numbers on the top row instead of adding them.

### Worked Example

$$\frac{1}{4} - \frac{5}{7}$$

$$\begin{aligned}\text{Find common denominator: } & \frac{(1 \times 7)}{(4 \times 7)} - \frac{(5 \times 4)}{(4 \times 7)} \\ & = \frac{7}{28} - \frac{20}{28} \\ & = -\frac{13}{28}\end{aligned}$$

## Simplifying Fractions

Sometimes it is possible to simplify a fraction and this is done by dividing the top and the bottom number by the same number until they won't be divided by the same number any more.

### WORKED EXAMPLE

Simplify the fraction  $\frac{18}{20}$

Both 18 and 20 divide by 2 to give:  $\frac{9}{10}$

As  $\frac{9}{10}$  can't be simplified further the fraction  $\frac{18}{20} = \frac{9}{10}$

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## QUESTIONS

Calculate the following giving your answer in its simplest form.

1)  $\frac{7}{9} \times \frac{1}{2}$

2)  $\frac{12}{16} \times \frac{3}{4}$

3)  $\frac{3}{7} \mid \frac{5}{7}$

4)  $\frac{11}{15} + \frac{3}{6}$

5)  $\frac{14}{17} - \frac{4}{5}$

6)  $(\frac{2}{3} \times \frac{2}{9}) \mid (\frac{6}{11} \times \frac{1}{5})$

# FACTORS, MULTIPLES AND PRIME FACTORS OF NUMBERS

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## 7

When asked to find the factors of a number, you simply find all the numbers which divide into it.

### WORKED EXAMPLE

Find the factors of 30.

The method is to first find the smallest number which divides into the thirty and continue to do this with bigger numbers until they start to repeat.

i.e. First do  $30/1 = 30$

Therefore thirty and one are multiples of thirty.

Now do  $30/2 = 15$

Therefore two and fifteen are also multiples of thirty.

Next do  $30/3 = 10$

Therefore three and ten are also multiples of thirty.

NB: As thirty will not divide by four exactly you should move straight onto five.

$30/5 = 6$

Therefore the final factors of thirty are five and six.

Factors of thirty: 1, 2, 3, 5, 6, 10, 15, 30.

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When asked to find the multiples of a number you are only required to multiply the number by other numbers.

### WORKED EXAMPLE

Find the first three multiples of five.

$$5 \times 1 = 5$$

$$5 \times 2 = 10$$

$$5 \times 3 = 15$$

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When asked to express a number as a product of prime factors you write the smallest prime numbers that divide into it.

### WORKED EXAMPLE

Express twelve as a product of prime numbers.

The first smallest prime number that will divide into twelve is two:  $12/2 = 6$

Now the smallest prime number that divides into six is also two:  $6/2 = 3$

The smallest prime number that divides into three is three:  $3/3 = 1$

As you have now reached one you have found the prime factors and the answer should now be expressed as shown.

As a product of prime factors  $12 = 2 \times 2 \times 3$

# FACTORS, MULTIPLES AND PRIME

## FACTORS OF NUMBERS

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8

To find the lowest common multiple (LCM) you must multiply the numbers given until a common number is found. The worked example will explain how.

### WORKED EXAMPLE

Find the lowest common multiple of the numbers four and five.

To do this first write down some multiples of four and then some multiples of five as shown:

Multiples of four: 4, 8, 12, 16.

Multiples of five: 5, 10, 15, 20.

From the lists above it can be seen that there has not yet been a common number, therefore you continue to write down some more multiples of four and then five if necessary.

Further multiples of four: 20, 24. It can now be seen that the number 20 has appeared in both lists and is therefore the lowest common multiple of five and four.

To find the highest common factor (HCF) of two given numbers you write down the factors for both numbers and then compare the lists to find the highest number in common to both.

### WORKED EXAMPLE

Find the highest common factor of the numbers 25 and 30.

First write down the factors of 25: 1, 5, 25.

Now write down the common factors of 30: 1, 2, 3, 5, 6, 10, 15, 30.

By comparison of the factors for the numbers 25 and 30 it can be seen the highest number common to both is 5.

HCF of 25 and 30 = 5

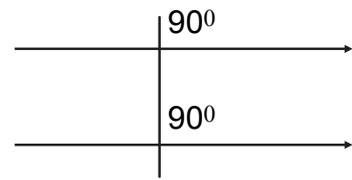
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### QUESTIONS

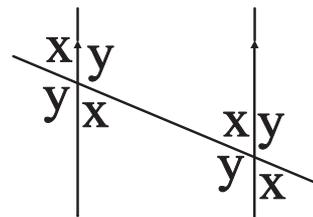
- 1) Write down three factors of twelve.
- 2) Write down the first six multiples of seven.
- 3) Express fifteen as a product of prime factors.
- 4) Write down all the multiples of nine which are less than ninety.
- 5) Find all the factors of twenty four.
- 6) For the number thirty write down:
  - a) the first three multiples
  - b) all the factors
  - c) and express thirty as a product of prime numbers.
- 7) Write down the first six multiples of the numbers three and fifteen; hence find the lowest common multiple of three and fifteen.
- 8) For the numbers twelve and eighteen write down all the factors and find the highest common factor.

Geometry problems can be solved easily as long as you learn the few facts which are sometimes needed; these are shown below.

- 1) The first rule is that all the angles on a straight line ALWAYS add up to 180 degrees.
- 2) The angles in a triangle, whether it is right-angled or not, also add up to 180 degrees.
- 3) Triangles which are isosceles have two lengths which are equal and therefore two angles which are equal.
- 4) The angles in a quadrilateral, a four sided shape, add up to 360 degrees.
- 5) Angles around a particular point also equal 360 degrees.
- 6) When a line crosses two other lines at right angles the lines are always parallel.



- 7) When parallel lines are crossed by another line the angles opposite each other, on either side of the parallel lines are equal.
- 8) The above diagram also shows that the angles in the Z are equal.



### WORKED EXAMPLES

- 1) An isosceles triangle has two angles of 45 degrees, find the missing angle.

To do this you use the rule that angles in a triangle add up to 180 degrees.

Therefore you can take two angles of 45 degrees from 180 degrees to find the missing angle:

$$180 - 45 - 45 = 90$$

Missing angle = 90 degrees.

- 2) Three angles around a point add up to 275 degrees, find the missing angle.

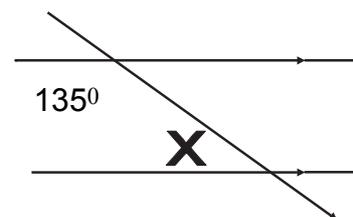
To do this you use the rule that angles around a point add up to 360 degrees.

Therefore you do the following:

$$360 - 275 = 85 \text{ degrees.}$$

- 3) Using the diagram below find angle 'x'.

Use the rule that when parallel lines are crossed by another line the angles opposite are equal and the angles in the Z are equal. Also use the rule that angles around a point add up to 360 degrees.



Therefore you do the following:

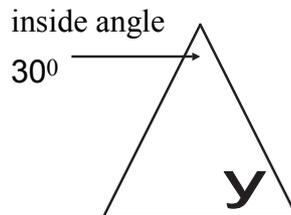
$$360 - 135 - 135 = 90 \text{ degrees}$$

As 90 degrees is equal to two angle 'x' you divide it by two to find the value of angle 'x'.

$$90 \div 2 = 45 \text{ degrees.}$$

$$x = 45 \text{ degrees.}$$

4) In the isosceles triangle shown below find the value of the unknown angle 'y'.

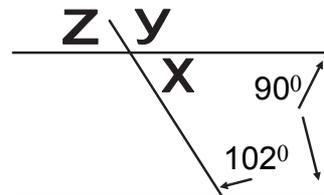


$$180 - 30 = 150 \text{ degrees}$$

$$150 \div 2 = 75 \text{ degrees}$$

## QUESTIONS

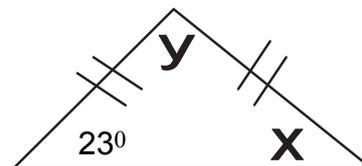
1) In the diagram shown find the angles labelled x, y, and z by using the rules learnt in this section.



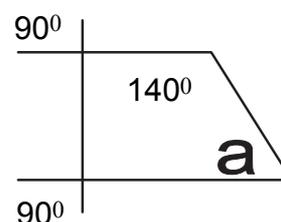
2) If a quadrilateral has angles of 30 degrees, 14 degrees and 150 degrees find the value of the unknown angle.

3) An angle in a straight line equals 50 degrees and the remaining two angles are equal in value, find the value of one of the unknown angles on the straight line.

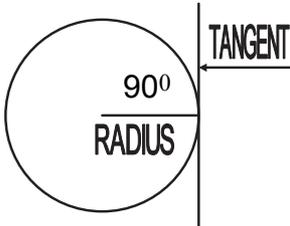
4) For the diagram shown opposite find the value of the angles labelled x and y, given that the triangle is isosceles.



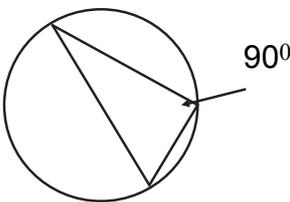
5) Using the information provided in the diagram opposite, find the value of the angle labelled 'a' by showing clearly the method used.



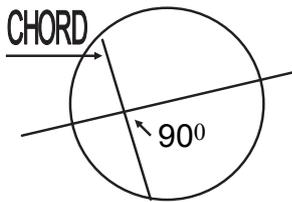
Geometry problems can also involve finding angles which are related to a circle and again these can be found easily as long as the few simple rules below are learnt.



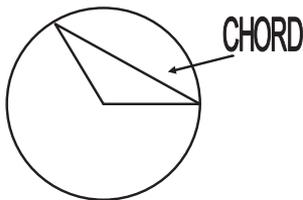
If a line touches a circle at one point it is said to be a tangent; and tangents are always at 90 degrees to the radius of the circle. This is shown in the diagram:



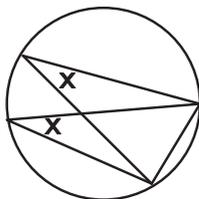
When a triangle is drawn inside a circle and has a base length which is equal to the diameter of the circle; the angle of the triangle which touches the circumference of the circle is always 90 degrees.



Any line which is drawn across a circle at any point in the circle is said to be a chord. A line cutting the chord at 90 degrees is an angle bisector and must be the diameter of the circle.

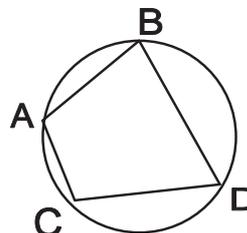


When two radii are drawn in a circle and are crossed by a chord the triangle formed is an isosceles triangle. Therefore there are two angles which are equal and two lengths which are equal ( in this case the radii).

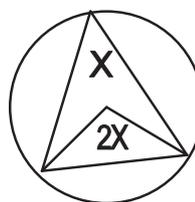


A chord drawn across a circle divides the circle into two segments. Triangles drawn in the same segment, which have the chord as their base length, have an angle which is of the same value at the point where the angle touches the circumference of the circle.

If a quadrilateral is drawn in a circle so that the corners of the quadrilateral touch the circumference at one point the quadrilateral is said to be a cyclic quadrilateral. The angles which are opposite each other in a cyclic quadrilateral always add up to 180 degrees.



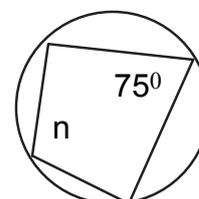
When two triangles are drawn so that they both have the chord of a circle as their base length; so that one triangle forms an angle at the centre of the circle and the other triangle forms an angle at a point on the circumference of the circle, the angle at the centre is always double the angle at the circumference. This is shown in the diagram below:



## WORKED EXAMPLES

1) In the diagram shown opposite write down the value of 'n'.

This involves our knowledge of cyclic quadrilaterals: angles opposite each other in a cyclic quadrilateral equal 180 degrees. Angle 'n' is opposite an angle of value 75 degrees therefore we do the following:



$$180 - 75 = 105 \text{ degrees}$$

$$\text{Angle 'n'} = 105^\circ \text{ degrees}$$

2) Using the information provided in the diagram write down the value of angle 'p' given that the angle at the centre equals 55 degrees. Also calculate the value of angle 'q' to three significant figures.

It can be seen that the triangle including angle 'p' is an isosceles triangle and as the angle at the centre equals 55 degrees we use the following method as the two unknown angles are equal and angles in a triangle add up to 180 degrees.

$$180 - 55 = 125 \text{ degrees}$$

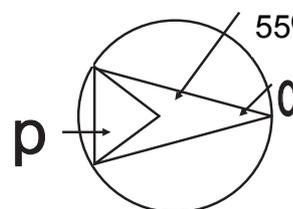
$$125 \div 2 = 62.5 \text{ degrees}$$

$$\text{Angle 'p'} = 62.5^\circ \text{ degrees.}$$

To find angle 'q' we use the rule that the angle at the centre is twice the angle at the edge:

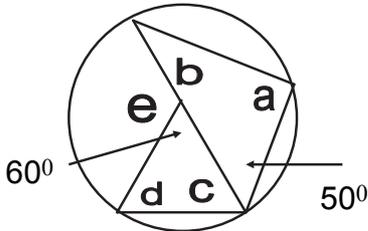
$$55 \div 2 = 27.5 \text{ degrees}$$

$$\text{Angle 'q'} = 27.5^\circ \text{ degrees.}$$

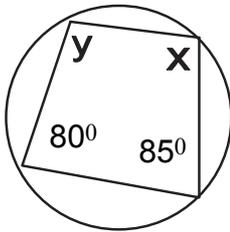


## QUESTIONS

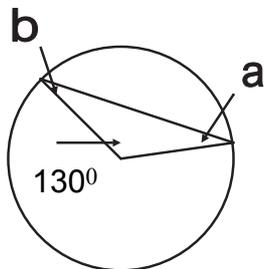
1) Find the value of each of the missing angles in the diagram below.



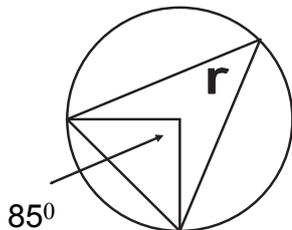
2) Find the value of the angle marked  $x$  and the angle marked  $y$  in the diagram below.



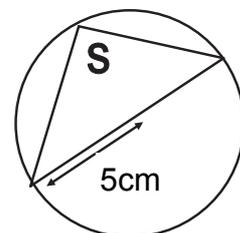
3) Explain why the angles labelled 'a' and 'b' must be equal, and find their values.



4) Find the value of angle 'r' in the diagram below.



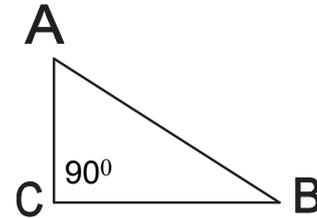
5) Write down the value of angle 's' and calculate the length of the diameter of the circle given that half the base length of the triangle is 5cm.



# PYTHAGORAS' THEOREM

14

Pythagoras' Theorem allows us to find a missing length in a right-angled triangle.



The diagram above shows how the theorem is applied to find the missing length AB. It can be seen from the diagram that the right-angle in the triangle is opposite the length AB and an important fact that should be learned is:

- the longest length in a right angled triangle always faces the right-angle.

Using Pythagoras' theorem to find the longest length (AB) the equation is:

$$AB^2 = AC^2 + BC^2$$

However if the length opposite the right angle is known (the longest length) and one of the other lengths is known the equation is rearranged to find the value of the unknown length:

$$AC^2 = AB^2 - BC^2$$

## WORKED EXAMPLES

1) For the triangle shown below find the length BC.

The length BC is opposite the right-angle so the equation used is:

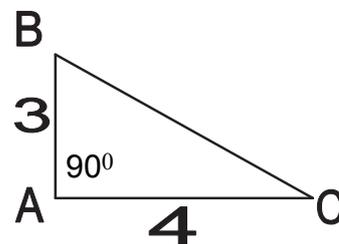
$$BC^2 = AC^2 + AB^2$$

$$BC^2 = 4^2 + 3^2$$

$$BC^2 = 25$$

$$BC = \sqrt{25}$$

$$BC = 5\text{mm.}$$



2) For the triangle shown find the length X.

The length X is not opposite the right angle so the equation to find X is to take the longest length squared from the shortest length squared:

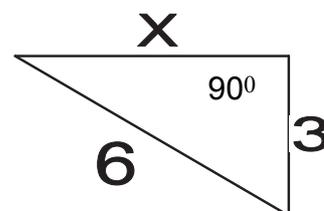
$$X^2 = 6^2 - 3^2$$

$$X^2 = 36 - 9$$

$$X^2 = 27$$

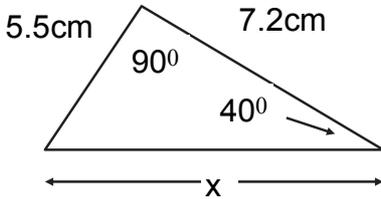
$$X = \sqrt{27}$$

$$X = 5.2 \text{ cm (to 1 decimal place)}$$

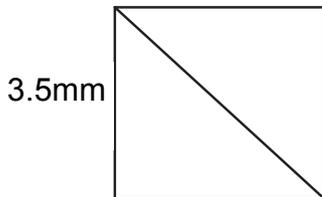


# PYTHAGORAS' THEOREM

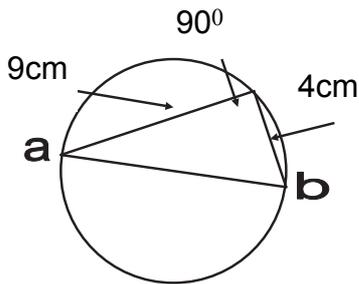
## QUESTIONS



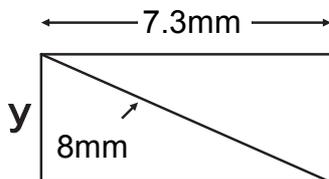
- 1) For the triangle opposite find:  
a) the value of the length 'x' to three significant figures  
b) the value of the missing angle.



- 2) The square shown has a diagonal which cuts through it, find the length of this diagonal to three significant figures.



- 3) Find the value of the length AB to three significant figures.

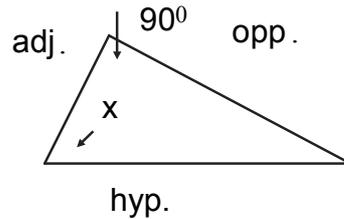


- 4) A rectangle has a diagonal passing through it of length 8mm, find the value of the side labelled 'y'.

Trigonometry questions involve finding either the value of an angle in a right-angled triangle given two appropriate lengths, or the length of a side being given the value of the appropriate angle and a length. However before attempting any trigonometry questions the following few phrases and equations should be learnt.

SOHCAHTOA is a phrase which can help you remember the following equations:

- 1) Sin 'x' = opposite ÷ hypotenuse
- 2) Cos 'x' = adjacent ÷ hypotenuse
- 3) Tan 'x' = opposite ÷ adjacent



Hypotenuse is the side opposite the right angle.

Opposite is the side facing the angle being used in the equation.

Adjacent is the side which is next to the angle being used in the triangle.

## WORKED EXAMPLES

- 1) For the triangle below find the value of the unknown angle 'y'.

We have been provided with the value of the opposite and the hypotenuse and can therefore use:

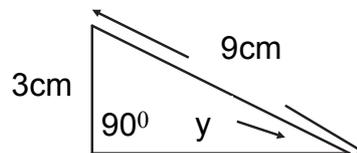
$$\sin y = \text{opposite} / \text{hypotenuse.}$$

$$\sin y = 3/9$$

$$\sin y = 1/3$$

We now do inverse  $\sin^{-1} 1/3$  to find the value of y:

$$y = 19.5^\circ \text{ degrees (to 1 decimal place)}$$



- 2) For the triangle below find the value of the length labelled 'x'.

We have been provided with the value of the angle adjacent to the unknown length and the value of the hypotenuse; we therefore can rearrange the following equation to find 'x'.

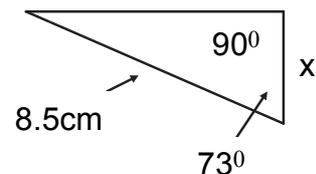
$$\cos x = \text{adjacent} / \text{hypotenuse}$$

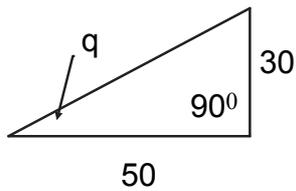
$$\text{Adjacent} = \cos x \times \text{hypotenuse}$$

$$\text{Adjacent} = \cos 73 \times 8.5$$

$$\text{Adjacent} = 2.49$$

$$x = 2.49 \text{ cm (to two decimal places)}$$





3) Using the information in the triangle opposite find the value of angle 'q' to one decimal place.

The diagram shows that we have the value of the length opposite the unknown angle and the value of the length adjacent to the unknown angle so we use the equation:

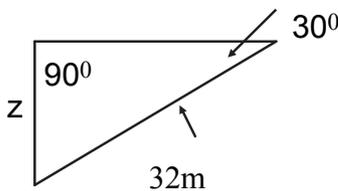
$$\tan q = \text{opposite} / \text{adjacent}$$

$$\tan q = 30 / 50$$

$$\tan q = 0.6$$

Now do inverse tan 0.6 on your calculator to give:

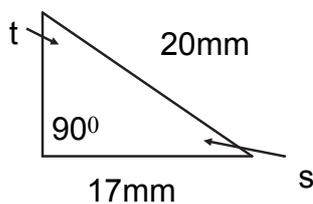
$$q = 31.0^\circ \text{ (to 1 decimal place)}$$



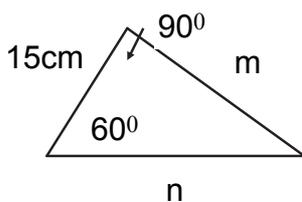
## QUESTIONS

1) For the triangle opposite find the length labelled 'z' by rearranging the equation:

$$\sin z = \text{opposite} \div \text{hypotenuse.}$$



2) For the right angled triangle find the value of angle 's' by using the cos rule, hence find the value of 't'.



3) Find the length labelled 'm' in the triangle by using the tan rule, hence find the length 'n' by using Pythagoras' theorem.

# APPLICATION OF THE SINE RULE

18

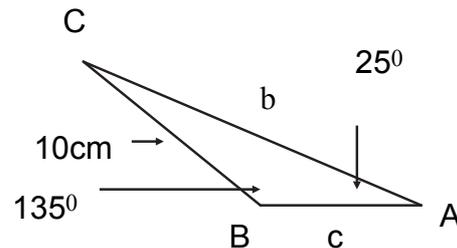
The Sine rule:

$$a \div \sin A = b \div \sin B$$

This rule can be applied to triangles which don't have a right angle in them to find the value of an unknown length when given the corresponding angle and the value of another length in the triangle and its corresponding angle.

## WORKED EXAMPLE

For the triangle opposite calculate the value of length 'b'.



Before you attempt this question you should understand that the side of the triangle opposite to an angle are corresponding.

You should now substitute the values you have been given in to the Sine rule equation:

$$b \div \sin 135 = 10 \div \sin 25$$

To find 'b' you now rearrange the equation so that it reads:

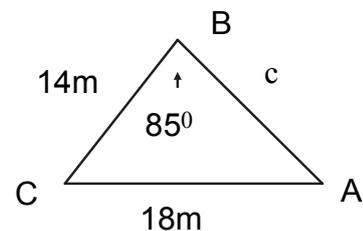
$$b = (10 \times \sin 135) \div \sin 25$$

$$b = 16.73 \text{ cm (to 2 decimal places)}$$

The Sine rule can also be applied to triangles to find the value of an unknown angle when given the value of its corresponding side and the values of another angle and its corresponding side.

## WORKED EXAMPLE

Using the information in the triangle opposite calculate the value of angle A.



To find the value of an unknown angle the Sine rule is always rearranged so that it reads:

$$\sin A \div a = \sin B \div b$$

$$\sin A \div 14 = \sin 85 \div 18$$

Rearrange the equation to find Sin A:

$$\sin A = (14 \times \sin 85) \div 18$$

$$\sin A = 0.7748$$

On your calculator now do inverse sine 0.7748 to find the value of angle A.

$$A = 50.8^\circ \text{ (to 1 decimal place)}$$

# APPLICATION OF THE COSINE RULE

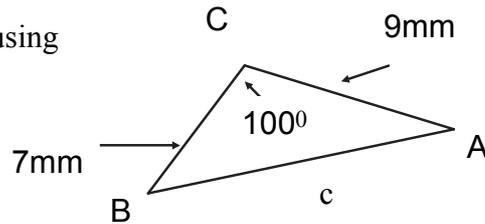
The Cosine rule:

$$a^2 = b^2 + c^2 - (2 \times b \times c \times \cos A)$$

This rule can again be applied to all triangles to find the value of an unknown length given its corresponding angle and the value of the two other lengths in the triangle.

## WORKED EXAMPLE

For the triangle ABC find the length of side 'c' using the cosine rule.



You first rearrange the equation so that it reads:

$$c^2 = a^2 + b^2 - (2 \times a \times b \times \cos C)$$

Substitute the appropriate values into the equation:

$$c^2 = 7^2 + 9^2 - (2 \times 7 \times 9 \times \cos 100)$$

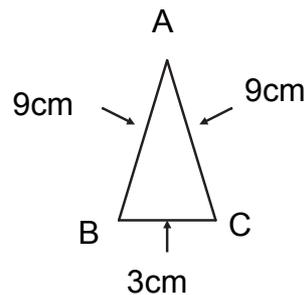
$$c^2 = 151.9$$

$$c = 12.32 \text{ mm (to 2 decimal places)}$$

The Cosine rule can also be applied to any triangle to find the value of an unknown angle given the value of all three lengths in the triangle.

## WORKED EXAMPLE

For the triangle ABC find the value of angle A.



First rearrange the Cosine rule:

$$\cos A = (b^2 + c^2 - a^2) / (2 \times b \times c)$$

$$\cos A = (9^2 + 9^2 - 3^2) / (2 \times 9 \times 9)$$

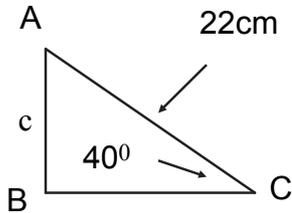
$$\cos A = 153 \div 162$$

$$\cos A = 0.9444$$

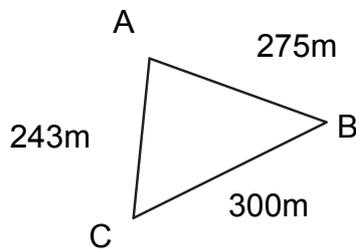
On your calculator do inverse cos 0.9444 to find the value of angle A.

$$A = 19.2^\circ \text{ (to 1 decimal place)}$$

## QUESTIONS

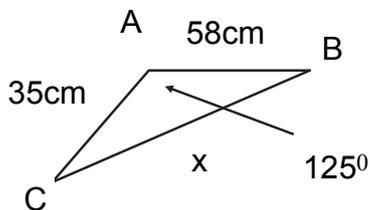


1) Use the sine rule to find the length 'c' in the triangle.



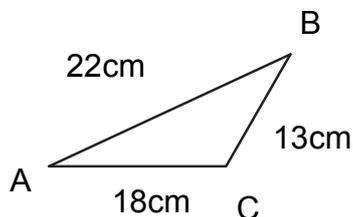
2) Using the information given in the triangle below find the value of:

- angle B using the cosine rule
- angle C using the sine rule
- angle A.



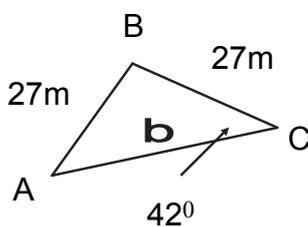
3) Use the cosine rule to find the length 'x' in the triangle.

4) For the same triangle, using the length 'x' found, use the sine rule to find the value of angle C.



5) For the triangle calculate:

- Angle C
- Angle A
- Angle B



6) The triangle opposite is an isosceles triangle, find:

- Angle B
- Length 'b' to three significant figures.

# ANSWER PAGE

# 21

Page 2:

- 2) Equilateral triangle;  $60^\circ$
- 3) 4 lines of symmetry
- 4) Number of sides =  $360^\circ$ / External angle
- 5) 7 sides
- 6a)  $140^\circ$    b) 9 sides

Page 4:

- 1)  $787.5 \text{ cm}^2$
- 2)  $78.5 \text{ cm}^2$
- 3)  $3 \frac{11}{18} \text{ cm}^2$
- 4)  $180 \text{ m}^2$
- 5)  $2036 \text{ cm}^3$  (to 4 sig.fig.)
- 6a)  $4 \text{ cm}^2$    b)  $16 \text{ cm}^3$
- 7)  $2572 \text{ cm}^3$  (to 4 sig.fig.)

Page 6:

- 1)  $\frac{7}{18}$
- 2)  $\frac{9}{16}$
- 3)  $\frac{3}{5}$
- 4)  $1 \frac{7}{30}$
- 5)  $\frac{2}{85}$
- 6)  $1 \frac{29}{81}$

Page 8:

- 1) 1, 2, 3
- 2) 7, 14, 21, 28, 35, 42
- 3)  $15 = 3 \times 5$
- 4) 9, 18, 27, 36, 45, 54, 63, 72, 81
- 5) 1, 2, 3, 4, 6, 8, 12, 24
- 6a) 30, 60, 90
- b) 1, 2, 3, 5, 6, 10, 15, 30
- c)  $30 = 2 \times 3 \times 5$
- 7) For 3: 3, 6, 9, 12, 15, 18  
For 15: 15, 30, 45, 60, 75, 90  
LCM = 15
- 8) For 12: 1, 2, 3, 4, 6, 12  
For 18: 1, 2, 3, 6, 9, 18  
HCF = 6

Page 10:

- 1)  $x = 78^\circ, z = 78^\circ, y = 102^\circ$
- 2)  $166^\circ$
- 3)  $65^\circ$
- 4)  $x = 23^\circ, y = 134^\circ$
- 5)  $a = 40^\circ$

Page 13:

- 1)  $a = 90^\circ, b = 40^\circ, c = 60^\circ, d = 60^\circ, e = 120^\circ$
- 2)  $x = 100^\circ, y = 95^\circ$
- 3) The triangle is isosceles as two of the sides of the triangle are the radii of the circle so angle A and angle B must be equal.  
 $a = 25^\circ, b = 25^\circ$
- 4)  $r = 42.5^\circ$
- 5)  $s = 90^\circ$ , diameter = 10cm

Page 15:

- 1a) 9.06 cm b)  $50^\circ$
- 2) 4.95 mm
- 3) AB = 9.85 cm
- 4)  $y = 3.27 \text{ mm}$

Page 17:

- 1)  $z = 16 \text{ m}$
- 2)  $s = 31.8^\circ, t = 58.2^\circ$
- 3)  $m = 26.0 \text{ cm}$  (to 1 decimal place)  
 $n = 30 \text{ cm}$

Page 20:

- 1)  $c = 14.14 \text{ cm}$  (to 2 decimal places)
- 2a)  $B = 49.8^\circ$  (to 1 d.p.)
- 2b)  $C = 59.8^\circ$  (to 1 d.p.)
- 2c)  $A = 70.4$
- 3)  $a^2 = b^2 + c^2 - (2bc \times \cos A)$   
 $x = 83.17 \text{ cm}$  (to 2 d.p.)
- 4)  $C = 34.8^\circ$  (to 1 d.p.)
- 5a)  $C = 88.9^\circ$  (to 1 d.p.)
- 5b)  $A = 36.2^\circ$  (to 1 d.p.)
- 5c)  $B = 54.9^\circ$  (to 1 d.p.)
- 6a)  $B = 96^\circ$
- 6b)  $b = 40.13 \text{ m}$