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NOTE: All diagrams are deliberately approximate. This is to ensure that the answers cannot be easily calculated by simply measuring the lengths and angles from the diagrams, and also to test the students' ability to accurately draw diagrams from given information.

MATHEMATICS REVISION PACK 2

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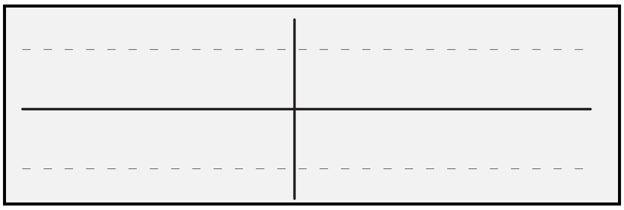
SIN, COS AND TAN GRAPHS 1

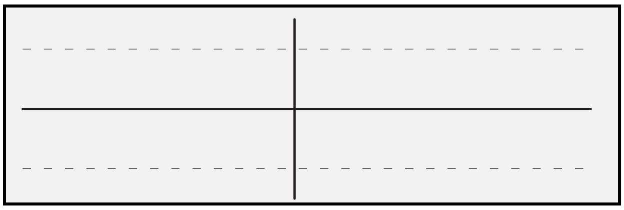
The GCSE syllabus requires you to know the Y = SIN X , Y = COS X and the Y = TAN X graphs so that they can be applied to questions. Find out about and draw the following:

Graph of Y = SIN X



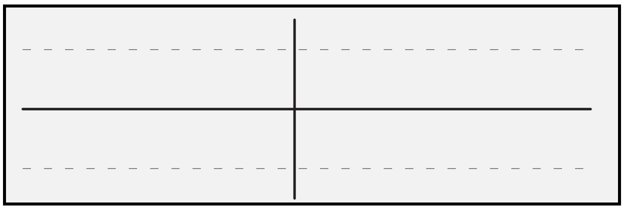
This graph has y-limits of positive one and negative one and the graph repeats itself every 360^{0}.

Graph of Y = COS X



This graph also has y-limits of positive one and negative one. It is also repeated every 360^{0}.

Graph of Y = TAN X



This graph does not have y-limits of positive one and negative one but goes to positive infinity and negative infinity on either side of the asymptotes. The graph repeats every 180^{0}.

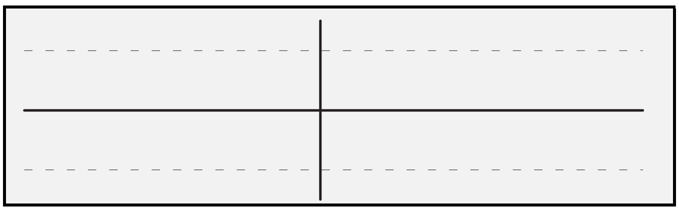
WORKED EXAMPLE

For the graph of Y = SIN X where X ranges from -360^{0} to 360^{0} find all the values of X when SIN X = 0.75.

First sketch the graph for the range given in the question.

Then draw the horizontal line sin x = 0.75 across the graph.

Vertical lines should now be drawn from the point at which the y = sin x graph and the line sin x = 0.75 intersect to the x-axis.



Using your calculator you can find one of the values of X by doing the inverse sin of 0.75:

$\sin^{-1} 0.75 = 48.6^{0}$ (to one decimal place). Therefore $X = 48.6^{0}$ (to one decimal place)

To find the other three values of X you use the symmetry of the graph.

It can be seen from the graphs symmetry that the distance between the second X value and 180^0 is also 48.59^0. Therefore the second X value can be found by performing the calculation:

$180^0 - 48.59^0 = 131.4^0$ (to one decimal place).

The third value of X can be found by:

$-180^0 - 48.59^0 = -228.6^0$ (to one decimal place).

The fourth value of X can now be found by:

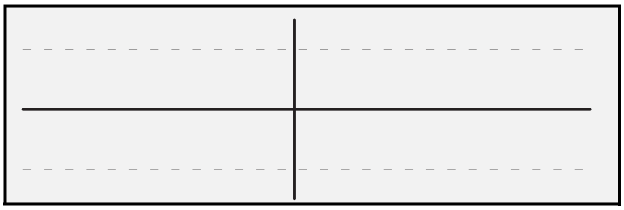
$-360^0 + 48.59^0 = -311.4^0$ (to one decimal place).

Therefore the values of X when SIN X = 0.75 are 48.6^0, 131.4^0, -228.6^0 and -311.4^0.

WORKED EXAMPLE

For the graph of Y = COS X where X ranges from -90^0 to 360^0 find all the values of X when COS X = -0.5.

First sketch the graph for the range given in the question. Then draw the horizontal line cos x = -0.5 across the graph. Vertical lines should now be drawn from the point at which the y = cos x graph and the line cos x = -0.5 intersect to the x-axis.



Using your calculator you can find one value of X by doing the inverse cos of -0.5:

$\cos^{-1} -0.5 = 120^0$.

The second value of X can now be found by using the symmetry of the graph and performing the calculation:

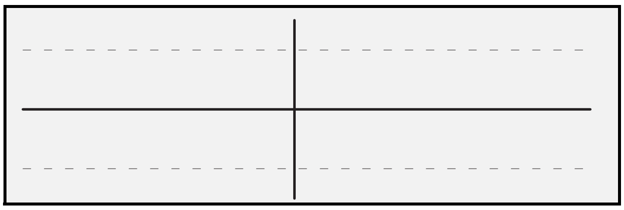
$360^0 - 120^0 = 240^0$.

Therefore the values of X when COS X = -0.5 are 120^0 and 240^0.

WORKED EXAMPLE

For the graph of Y = TAN X where X ranges from -180^0 to 180^0 find all the values of X when TAN X = 0.25.

First sketch the graph for the range given in the question. Then draw the horizontal line tan x = 0.25 across the graph. Vertical lines should now be drawn from the point at which the graph y = tan x intersects the line tan x = 0.25 to the x-axis.



Using your calculator you can find one of the values of X by doing the inverse tan of 0.25:

$\tan^{-1} 0.25 = 14.03^0$ (to 2 decimal places).

To find the other value of X you use the symmetry of the graph:

$-180^0 + 14.03^0 = -165.96^0$ (to 2 decimal places).

Therefore the values of X when TAN X = 0.25 are 14.0^0 and -165.7^0 (to 1 decimal place).

QUESTIONS ON SIN, COS & TAN GRAPHS

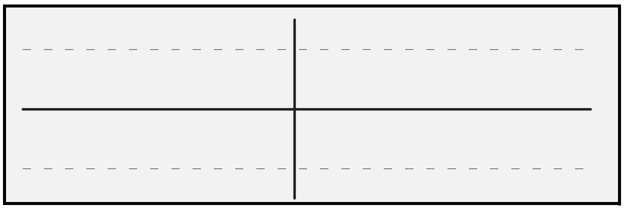
3

1) Sketch the graph of Y = COS X where X ranges from -180^0 to 450^0, clearly showing the y-limits.

2a) Sketch the graph of Y = SIN X where X ranges from 0^0 to 360^0.

2b) Using the graph sketched in part (a) find all the values of X when SIN X = -0.3.

3) Sketch the graph below of Y = TAN X where X ranges from -130^0 to 260^0. Find the values of X when TAN X = 2 showing your method clearly.



4a) Find the value of 'z' when sin 60^0 = z.

4b) For the graph Y = SIN X where X ranges from -180^0 to 360^0 find all the values of X when sin X = z.

5) Sketch the graph y = tan x when x ranges from -90^0 to 360^0 and state the y-limits and how often the graph repeats itself.

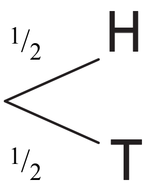
PROBABILITY 4

The easiest method of solving probability questions is to display the information in the form of a tree diagram. A tree diagram is very simple to draw and you must learn the following few facts:

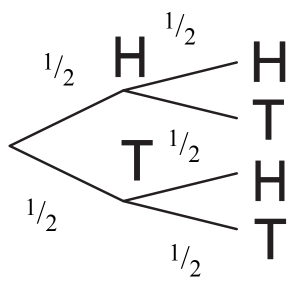
1) The sum of the probabilities on branches extending from one point should always equal one.

2) To calculate the final answer you multiply the probabilities along the branches.

For example if we draw a tree diagram to show the possible outcomes when two fair coins are tossed. First you consider the possible outcomes when one coin is tossed and they are a head or a tail. Both the head and the tail have an even chance of occurring therefore the head has a probability of 1/2 (i.e. a one in two chance of occurring) and the tail also has the probability of 1/2. Using the information you draw your first two branches of the tree along with the outcome and its associated probability as shown below:



You now consider the possible outcomes of tossing another fair coin and again they are a head and a tail; each one having the probability of 1/2 of occurring. It should also be noted that the outcome of the first coin has no effect on the outcome of the second coin. For example if the outcome of the first coin was a head the outcome of the second coin can be a head or a tail. You can therefore add the next set of branches onto your tree diagram showing the outcomes and their associated probabilities.



H = HEADS
T =TAILS

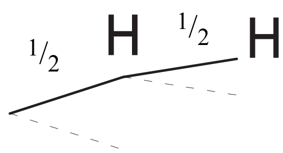
The tree diagram above can now be used to answer question such as:

1)Calculate the probability that when two coins are tossed both will show heads. To do this you write down the probability of obtaining one head from your tree diagram and multiply it by the probability of obtaining a second head:

Probability of 2 heads = $^1/_2 \times {}^1/_2$

$= {}^1/_4$

The part of the tree diagram used to solve this question is shown opposite.



2) Calculate the probability of obtaining one tail when two coins are tossed.

By looking at the tree diagram we can see that there are two possible routes in which a tail can occur:

A head on the first coin followed by a tail on the second coin.

A tail on the first coin followed by a head on the second coin.

As either one can occur we find the probabilities of each one occurring and then add the two values together:

Probability of a head then a tail = $^1/_2$ X $^1/_2$ = $^1/_4$

Probability of a tail then a head = $^1/_2$ X $^1/_2$ = $^1/_4$

Probability of 1 tail when 2 coins are tossed = $^1/_4$ + $^1/_4$

= $^1/_2$

3) Calculate the probability of obtaining at least one tail when two coins are tossed.

'At least' questions can all be solved very easily as the above question is asking us to find the probability of obtaining one tail or the probability of obtaining two tails. It is easier if we look at it as:

1 - probability of other outcomes = probability of obtaining at least one

i.e. 1 - probability of no tails = probability of obtaining at least one tail.

This is due to the fact that the sum of all the end result is equal to one and the only other possible outcome that can occur is for there to be no tails.

Probability of no tails = Probability of head on first coin X probability of head on second coin

= $^1/_2$ X $^1/_2$

=$^1/_4$

Probability of at least one tail = 1 - probability of no tails

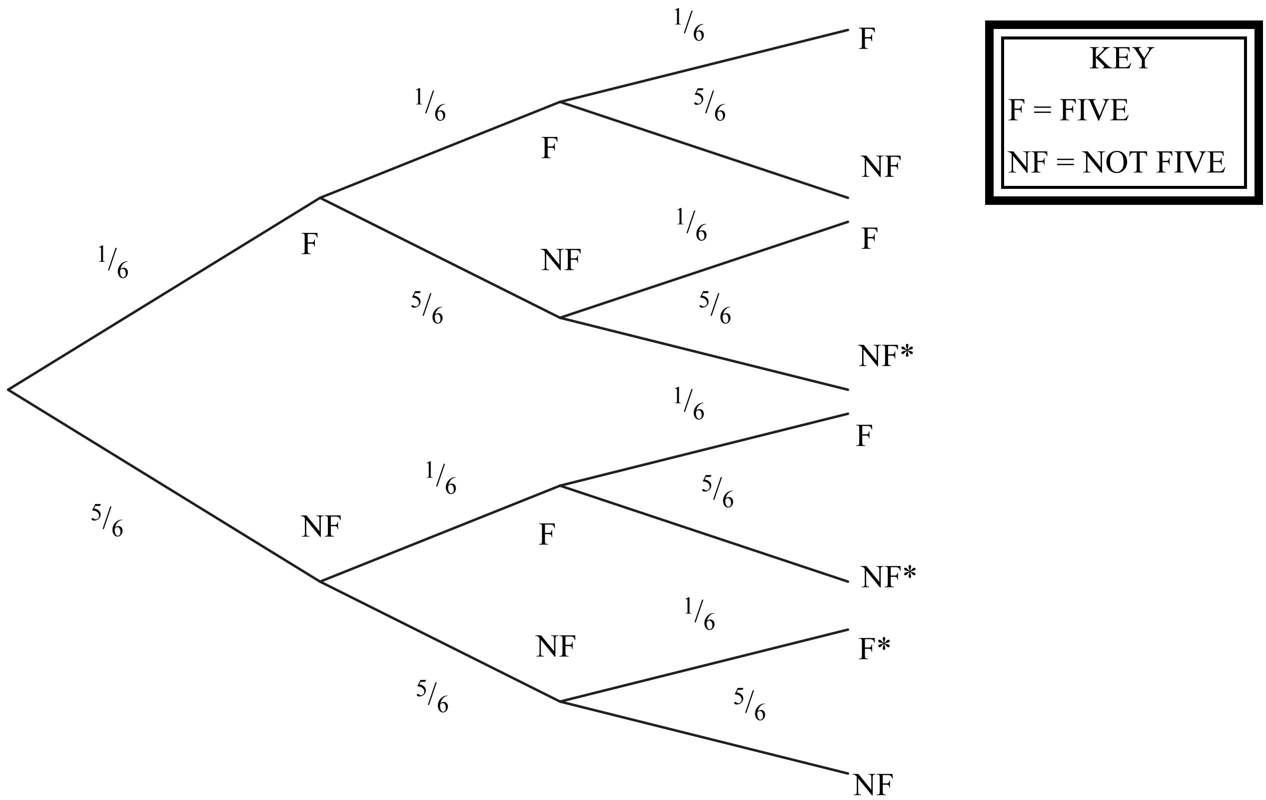
= 1 - $^1/_4$

= $^3/_4$

WORKED EXAMPLE

A fair dice is thrown three times, find the probability of obtaining one five.

First a tree diagram is drawn using the method used in the example above, however in this case the possible outcomes are a five or not a five occurring when the dice is thrown and the probabilities being $^{1}/_{6}$ and $^{5}/_{6}$ respectively. Again the previous outcome has no effect on the following outcomes.



From the tree diagram drawn we can see that there are three routes in which one five can occur and these routes have been highlighted with an asterisk*. We calculate the value of each route individually by multiplying the probabilities along the route and we then add the values obtained together for our final answer.

Probability of a five, not a five, not a five = $^{1}/_{6} \text{ X } ^{5}/_{6} \text{ X } ^{5}/_{6} = ^{25}/_{216}$

Probability of not a five, a five, not a five = $^{5}/_{6} \text{ X } ^{1}/_{6} \text{ X } ^{5}/_{6} = ^{25}/_{216}$

Probability of not a five, not a five, a five = $^{5}/_{6} \text{ X } ^{5}/_{6} \text{ X } ^{1}/_{6} = ^{25}/_{216}$

Probability of obtaining a 5 when a dice is tossed three times = $^{25}/_{216} + ^{25}/_{216} + ^{25}/_{216}$

$= ^{25}/_{72}$

QUESTIONS

1a) A bag contains two red balls and three green balls, draw a tree diagram to show the possible outcomes and their associated probabilities when two balls are drawn at random from the bag with replacement.

1bi) Using the tree diagram find the probability of drawing a green ball followed by a red ball.

1bii) Find the probability of obtaining at no red balls.

1biii) find the probability of obtaining at least one red ball.

2a) A boy has a pack of 52 playing cards and takes a card at random and then replaces it, he then does this again two more times draw a tree diagram to show the probability that he picks an ace or not ace.

2b) Use this tree diagram to find the probability of him picking three aces.

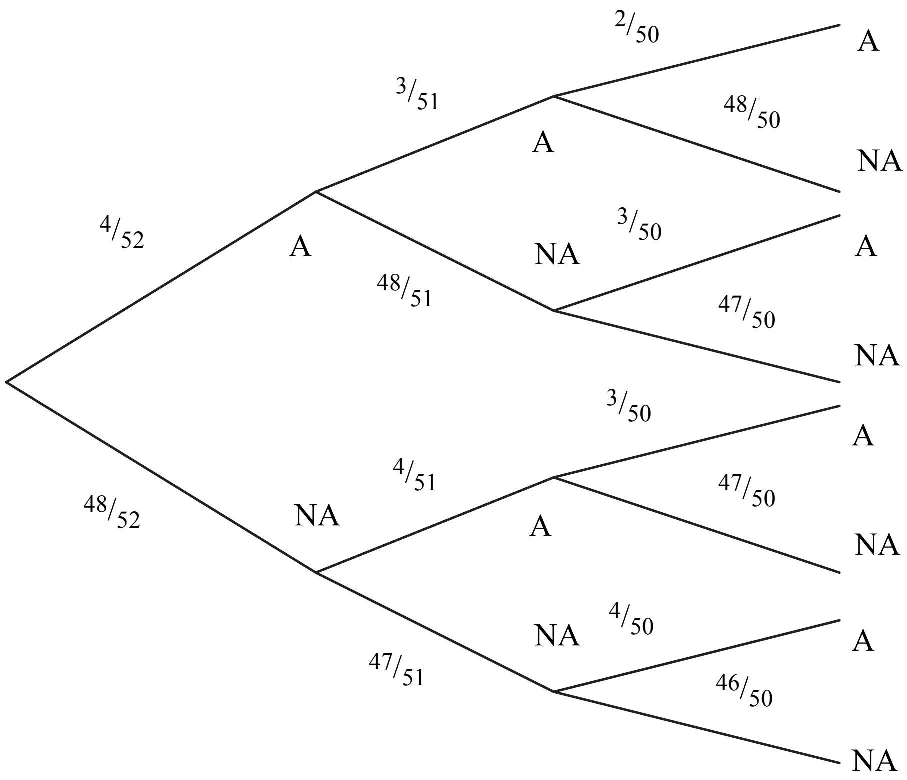
2c) Find the probability that he picks at least one ace.

3) The tree diagram drawn below shows the possible outcomes when three cards are picked at random from a pack of 52 without replacement.

3a) Find the probability of obtaining three aces.

3b) Find the probability of obtaining no aces.

3c) Find the probability of obtaining one ace.



KEY
A = ACE
NA = NOT ACE

Algebraic equations are a lot easier to solve than they look and you can tackle them as long as you understand the few basic rules.

SIMPLIFYING EQUATIONS INVOVING LIKE TERMS AND THEN SOLVING

For example consider the equation:

$3x^2 - 4 - 2x^2 = 0$

The first rule is that the equation contains separate terms - there are different types of terms in the equation above:

the x^2 term - eg. $3x^2$ and $-2x^2$ (called 'like terms')

the number -4 is also a term.

The second rule is that the like terms in the equation should be combined to simplify the equation:

$x^2 - 4 = 0$ (as $3x^2 - 2x^2 = 1x^2$)

This simplified equation can now be solved by first getting the x^2 term on its own by taking the four to the other side so that the equation reads:

$x^2 = 4$

You must remember that when you move any term to the other side of the equal sign, you do the opposite to what the term was initially doing. For the above example the number 4 initially had the negative sign in front of it but when it was moved to the other side it became positive four.

$x = 2$, or $x = -2$

(The rule above was also applied here as initially the x term was being squared so in order to get the x term on its own the opposite had to be done on both sides of the equal sign, which was to square root).

SIMPLIFYING EQUATIONS INVOLVING BRACKETS AND THEN SOLVING

For example consider the equation:

$3 (2x^2 -3) = 9$

The first rule is to simplify the equation by multiplying everything inside the brackets by everything outside the brackets:

$3 \times 2x^2 = 6x^2$

$3 \times -3 = -9$

Therefore the equation now reads:

$6x^2 - 9 = 9$

To solve the equation you must first get the $6x^2$ term on its own by taking the -9 term to the opposite side. As the term is negative nine to get it to the opposite side you must add nine so that the equation reads:

$6x^2 = 18$ $(18 = 9 + 9)$

To solve the equation to find x you must now divide both sides by six:

$6x^2 / 6 = x^2$

$18 / 6 = 3$

Therefore the equation reads: $x^2 = 3$

You must now square root both sides as again you do the opposite to what is being done to the x term: $x = \sqrt{3}$

Example 2

Consider the equation: $(2x + 3)(x - 1) = 0$

In cases where there are two sets of brackets and you are required to multiply the brackets out you have to multiply everything in the first bracket by everything in the second bracket. The steps below outline the procedure that leads to the final simplified equation.

1) Multiply the 2x in the first bracket with everything in the second bracket:

$2x \text{ X } x = 2x^2$

$2x \text{ X } -1 = -2x$

2) You now multiply the 3 in the first bracket with everything in the second bracket:

$3 \text{ X } x = 3x$

$3 \text{ X } -1 = -3$

Therefore the equation reads:

$2x^2 - 2x + 3x - 3 = 0$

3) You now collect the like terms to simplify the equation:

$2x^2 + x - 3 = 0$

This equation is therefore a quadratic equation and solving quadratic equations is discussed in the next topic.

SIMPLIFYING ALGEBRAIC FRACTIONS

You can simplify algebraic fractions by applying your knowledge of solving ordinary fractions as the principles are the same:

Multiplication: With ordinary fractions you multiply the top row of numbers together and then multiply the bottom row of numbers together so here you do the same except with letters.

WORKED EXAMPLE

$2a / 3b^2 \text{ X } 3b^3 / a = 6ab^3 / 3ab^2$

This answer can be further simplified by cancelling:

$6ab^3 / 3ab^2 = 2ab^3 / ab^2$ (here the six and the three have been divided by three)

$2ab^3 / ab^2 = 2b$ ('a' divides into 'a' exactly once and therefore cancels and 'b^2' divides into 'b^3' to give 'b').

Division: With ordinary fractions you simply invert the second fraction and then use the same rules as for multiplication.

WORKED EXAMPLE

Simplify:

$(25pq^2 / r^3) \mid (15q3 / r^4) = (25pq^2 / r^3)\ X\ (r^4 / 15q^3)$

$= 25pq^2r^4 / 15q^3r^3$

$= 5pq^2r^4 / 3q^3r^3$

$= 5pr / 3q$

Addition: Again you apply your knowledge of ordinary fractions when it comes to addition and that involves finding a common denominator first so that the letters and numbers on the bottom row are the same and the top rows can then be added. The easiest way to find a common denominator is to cross multiply which involves multiplying the top row of one fraction by the bottom row of another fraction.

WORKED EXAMPLE

Simplify:

$a^3 / 2a + 3/8 = 8a^3 / 16a + 6a / 16a$ [cross multiplication]

$= (8a^3 + 6a) / 16a$

Subtraction: This is the same as addition and involves finding the common denominator first and then subtracting the top row.

SIMPLIFYING EQUATIONS BY FACTORISATION

This basically involves taking out the highest common factor for all the terms in the equation e.g.

Factorise: $9z^2 + 15y^3 - 3z$

First you can see that there is a common factor of three that can be taken out:

$3 (3z^2 + 5y^3 - z)$

There are no other numbers or letters which are common to all the terms so the above equation is your final answer. However, for the second example below it is possible to take out common factors and numbers.

WORKED EXAMPLE

Factorise: $21q^4 + 7qr - 28qr^2$

First you should notice that there is a common factor of 7 in all the terms:

$7 (3q^4 + qr - 4qr^2)$

You should also notice that all the terms have the common letter 'q' so it can be taken out as a common factor:

$7q (3q^3 + r - 4r^2)$

Even though the letter 'r' is also in the equation it is only common to two of the terms and not all of them so it can not be taken out as a common factor. Therefore the factorised answer is:

$7q (3q^3 + r - 4r^2)$

ALGEBRAIC QUESTIONS

1) Simplify the expression: $2x^2 + 4x^3 - x^2 - 12$

2) Simplify the following equation and solve it to find 'x'.

$6x^3 - 3x^3 + 9 + 2x^2 = 90 + 2x^2$

3) Solve the following equation to find 'x' giving your answer to three significant figures.

$4(3x^2 - 6) = 0$

4) Express the given equation in a simplified form so that there are no brackets.

$(9x - 2)(3x + 4) = 0$

5) Simplify the following:

a) $4a / 6b^2 \; X \; 6b^3 / 2a$

b) $c / d^3 \; X \; cd^4 / bc$

c) $(rs^2 / p^3) \mid (rp / p^2 s)$

d) $de^2 / a + ca^2 / d$

e) $p / (p + r) - (r^2 + pr) / r$

6) Simplify the following equations:

a) $32r^3p^2 - 16rp^2 + 24r^2p = y$

b) $13xy^3 + 2x^2 - 17xy^3 + 2x^4 = z$

c) $12r^3t + 4rs^2 + 4r^4s = 0$

INEQUALITIES 12

Questions involving inequalities are very simple to solve as long as you apply your knowledge of algebra but you must understand the following symbols which appear in the equations:

$<$ This is the sign for "less than".

$>$ This is the sign for "greater than".

$\leq$ This is the sign for "less than or equal to".

$\geq$ This is the sign for "greater than or equal to".

In algebraic equations when you wish to take a negative term to the other side of the equal sign the term becomes positive and vice versa, it is the same with inequalities.

If you have for example: $4x = 16$ in algebra to find the value of 'x' you do the opposite of what is being done to the 'x' so in this case you would divide both sides by four; this is also true for inequalities.

However, if you are required to divide or multiply by a negative number the equality sign must be changed as shown in the example below:

$-5x \geq -20$ (This says -5x is greater than or equal to -20)

To find the value of x you have to divide both sides by -5 and as you are dividing both sides by negative number the sign changes from $\geq$ to $\leq$.

$x \leq {}^{-20}/_{-5}$

$x \leq 4$

WORKED EXAMPLES

1) Solve the inequality: $12x + 3 \leq 27$

First you must get the 12x on its own and to do that you must do the opposite to what is being done to it: you must take three from both sides as three is being added to the 12x.

$12x \leq 24$

To solve 'x' you do the opposite of what is being done to it, therefore you divide both sides by 12 as it is currently being multiplied by 12.

$x \leq 2$

2) Solve the inequality: $15x - 10 > 18x + 7$

You should first combine the like terms in this equation in order to simplify therefore you take 18x from both sides:

$-3x - 10 > 7$

You can then further simplify the above by adding ten to both sides as it is being taken from the -3x:

$-3x > 17$

To solve the inequality for 'x' you now have to divide both sides by -3 and as it is a negative number you must also remember to change the direction of the inequality sign:

$x < {}^{17}/_{3}$

$x < 5.666$

3) Solve the inequality: $x^2 - 3 \leq 33$

First you add three to both sides:

$x^2 \leq 36$

You now square root both sides to find the range of values in which x can lie as you must remember that when you square root a number you get a positive and a negative answer.

$x \leq 6$ and $x \geq -6$

These answers can now be combined and written as:

$-6 \leqq x \leqq 6$

4) Solve the inequality: $x^2 + 7 > 56$

First take seven from both sides:

$x^2 > 49$

Now square root both sides to find the values of x:

$x < -7$ and $x > 7$

In this case the inequalities can not be combined as the values of x are not continuous.

QUESTIONS

1) Give the meaning of the following inequality signs:

a) $>$

b) $\leq$

2) Solve the following inequalities giving non-integer answers to three significant figures:

a) $x^3 + 2 < 10$

b) $18y - 30 > 15 - 12y$

c) $2x^2 \leq 72$

d) $x^2 - 5 \geq 3x^2 - 15 + x^2$

e) $\sqrt{y} - 81 < 0$

3) Write down all the integer values of 's' which satisfy the inequality:

$s^2 + 6 \leq 22$

QUADRATIC EQUATIONS 14

Quadratic equations involve solving equations with an x^2 term and there are three methods in which it can be done. We will consider the equation:

$x^2 + 4x - 12 = 0$

We will solve the equation using the FACTORISATION METHOD which involves using brackets to solve the equation.

All quadratic equations can be expressed as: $ax^2 + bx + c$

In this case a = 1, b = 4, c = -12

We now start off by writing the following:

(x)(x) = 0

In order to fill in the brackets we must now find a pair of numbers which when added together equal four and when multiplied together equal minus twelve. Therefore we write down all the pairs of numbers which when multiplied together give -12.

-1 X 12 = -12	-3 X 4 = -12
1 X -12 = -12	3 X -4 = -12
-2 X 6 = -12	
2 X -6 = -12	

By looking at the pairs of numbers it becomes obvious that it is -2 and 6 as when they are added together they give four and when they are multiplied together they give minus twelve. Therefore we fill the brackets in as follows:

(x - 2)(x + 6) = 0

We can find the values of 'x' from this by making each bracket equal to 0 and rearranging the equation to find 'x'.

x - 2 = 0

x = 2 (add two to both sides)

x + 6 = 0

x = -6 (take six from both sides)

In this case the 'x' values are integers but that is not always the case so a second method can also be used to solve quadratic equations and it is by the use of the QUADRATIC FORMULA. The formula is:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] \mid 2a$

As long as you learn this formula solving quadratic equations becomes very simple as all you do is put the appropriate values into the equation. Consider the equation: $x^2 - 7x + 3 = 0$

In this case a = 1, b = -7, c = 3

Therefore put these values into the equation so that it reads:

$x = [- -7 \pm \sqrt{(-7^2 - 4 X 1 X 3)}] \div 2 X 1$

$x = [7 \pm \sqrt{37}] \div 2$

$x = 6.54$ or $x = 0.459$

The final method of solving quadratic equations is to use COMPLETING THE SQUARE and is a more complicated method but once you learn the few rules it is just as easy to apply. Consider the equation that we used in the factorisation method:

$x^2 + 4x -12 = 0$

First place the equation in a bracket as shown by halving the value of b (b = 4) and then square the bracket: $(x + 2)^2$

Then compare the above to the original equation and you will see that when you square two it equals four and to make it equal to minus twelve you need to take sixteen. Therefore the equation reads:

$(x + 2)^2 - 16 = 0$

To solve the equation you now add sixteen to both sides:

$(x + 2)^2 = 16$

You now square root both sides:

$x + 2 = 4$ and $x + 2 = -4$

Finally to find 'x' you take two from each side:

$x = 2$ and $x = -6$

QUESTIONS

1) Solve the following quadratic equations using factorisation.

a) $3x^2 + 7x + 2 = 0$

b) $6x^2 + 3x - 3 = 0$

c) $x^2 + 2x - 48 = 0$

d) $2x^2 - 2x - 10 = 2$

e) $5x^2 - 2x + 3 = 6x$

2) Solve the following quadratic equations using the completing the square method, giving non-integer answers to three significant figures.

a) $x^2 - 6x + 5 = 0$

b) $x^2 + 8x - 19 = 0$

c) $x^2 + 10x - 1 = 0$

d) State which of the above equations could also be solved by factorisation.

3a) $ax^2 + bx + c = 0$ Write down the quadratic formula used to solve this type of equation.

b) Solve the following equations using the quadratic formula:

i) $x^2 + 8x + 3 = 0$

ii) $4x^2 - 10x - 2 = 0$

SIMULTANEOUS EQUATIONS 16

When asked to solve simultaneous equations you are usually given two equations in which you have to find the values of an unknown. The example below will explain how to solve a pair of simultaneous equations.

WORKED EXAMPLE 1

Solve the following pair of simultaneous equations:

1) $3x - 12 = y$

2) $4y + 2x = 2$

First rearrange the equations above so that in both equations the 'x' and 'y' terms are on one side of the equal sign and the numbers are on the other side of the equal sign:

1) $3x - y = 12$

2) $2x + 4y = 2$

The coefficients in front of either the 'x' or the 'y' need to be equal for both equations. We will choose to make the coefficients in front of the 'y' the same by multiplying equation 1 by four:

1) $12x - 4y = 48$

2) $2x + 4y = 2$

We can now cancel the 'y' terms in both equations by adding equation 1 and equation 2 together as -4y and 4y cancel out when added:

(1) + (2)

$14x = 50$

To find x we now divide both sides by 14:

$x = 3.57$ (to three significant figures).

To find y we now substitute this value of 'x' into one of the equations.

$y = (3 \times 3.57) - 12$

$y = -1.29$ (to three significant figures)

WORKED EXAMPLE 2

Solve the following pair of the simultaneous equations:

1) $4x - 20 = 8y$

2) $6y + 30 = 8x$

First move the x and y terms to the same side of the equal sign and the numbers to the other side of the equal sign:

1) $4x - 8y = 20$

2) $-8x + 6y = -30$

We can now multiply equation 1 by two to make the x coefficients equal.

1) $8x - 16y = 40$

2) $-8x + 6y = -30$

We can now cancel the x terms in both equations by adding equations (1) and (2) together as -8x added to 8x equals 0.

(1) + (2)

$-10y = 10$

Divide both sides by -10 to find y.

$y = -1$

To find 'x' you substitute this value of y found into either equation one or two.

1) $4x - 20 = 8 X -1$

$4x - 20 = -8$

$4x = 12$

$x = 3$

WORKED EXAMPLE 3

Solve the following pair of simultaneous equations:

1) $9x + 12 = -2y$ 2) $y + 4 = 2x$

We rearrange the equations so that the 'x' and 'y' terms are on the same side.

1) $9x + 2y = -12$ 2) $-2x + y = -4$

We now can make the coefficients in front of the y equal by multiplying equation (2) by 2.

1) $9x + 2y = -12$ 2) $-4x + 2y = -8$

By taking equation (2) from equation (1) we can cancel the 'y' terms.

$13x = -4$

therefore $x = {}^{-4}/_{13}$

By substituting this value into either of the original equations we can find the value of y:

$y + 4 = 2x$

$y + 4 = {}^{-8}/_{13}$

$y = -4\ {}^{8}/_{13}$

QUESTIONS

1) Solve the following pair of of simultaneous equations giving exact answers:

a) $5x - y = 15$; $2y + 5 = x$

b) $2y + 2x = 3$; $6y + 3x = 0$

c) $x + 1 = y$; $7x = y - 3$

d) ${}^{1}/_{2}y + x = 22$; $y + 3x = 18$

VARIATION 18

In order to solve variation questions you must understand the following:

α this is the sign used for the phrase "is proportional to."

k this or any other letter is used to represent a constant in the equation.

EXAMPLES

If in a question you are told that 'm' is proportional to 'n' it is written as follows:

$m \ \alpha \ kn$ where k is a constant value.

If in a question it is given that 's' is inversely proportional to 'r' then it should be written as follows:

$s \ \alpha \ {}^{k}/_{r}$

More complicated questions may state:

'y' is proportional to the square of 'x' written as $y \ \alpha \ kx^2$

'g' is inversely proportional to the square root of 'h' written as $g \ \alpha \ {}^{k}/_{\sqrt{h}}$

WORKED EXAMPLE

1) The amount of electricity used by a radio is proportional to the cube of the length of time it is used for. If 0.5 units of electricity are needed when the radio is used for 30 minutes find how many units of electricity will be used when the radio is on for 45 minutes.

First we simplify the above as follows:

E this represents the number of units of electricity used.

T this represents the number of minutes the radio is on for.

From the phrase highlighted in the question we can form the following:

$E \ \alpha \ kT^3$ (k is a constant which has to be found)

The above proportionality can be written as the equation below and is simply formed by replacing the α with an = sign.

$E = kT^3$

We now input the data we have been given in the question and rearrange the equation to find the value of the constant, k.

$0.5 = k \times 30^3$

$0.5 = k \times 27000$

$k = 0.5 \mid 27000$

$k = 0.0000185$

Enter this value of 'k' into the original equation.

$E = 0.0000185 \times T^3$

By substituting the value given in the question the number of units of electricity can be solved.

$E = 0.0000185 \times 45^3$

$E = 1.686$ units (to 4 significant figures)

WORKED EXAMPLE 2

A can is filled with water and has a hole in its base; the volume of water the can holds is inversely proportional to the square of the time it is held for. When the water is held for 5 seconds the volume in the can is 100ml, find the volume of water held in the can after 10 seconds.

V this represents the volume of water in the can.

S this represents the time in seconds the water is held for.

From the data in the question we can form the following:

$V \propto k/S^2$

$V = k/S^2$

By substituting the values given we can find the value of 'k'.

$100 = k / 5^2$

$k = 100 \times 25$

$k = 2500$

Therefore the equation now reads:

$V = 2500 / S^2$

If we input the value of ten seconds into this equation we can now find how much water the can is holding.

$V = 2500 / 10^2$

$V = 2500/100$

$V = 25$ml

QUESTIONS

1) Write down the proportionality for the following phrases:

a) 'y' is inversely proportional to the cube root of x.

b) 'z' is directly proportional to the fourth power of 'r'.

c) 'm' is proportional to the square of 'o'.

d) 'q' is inversely proportional to the square root of 'p'

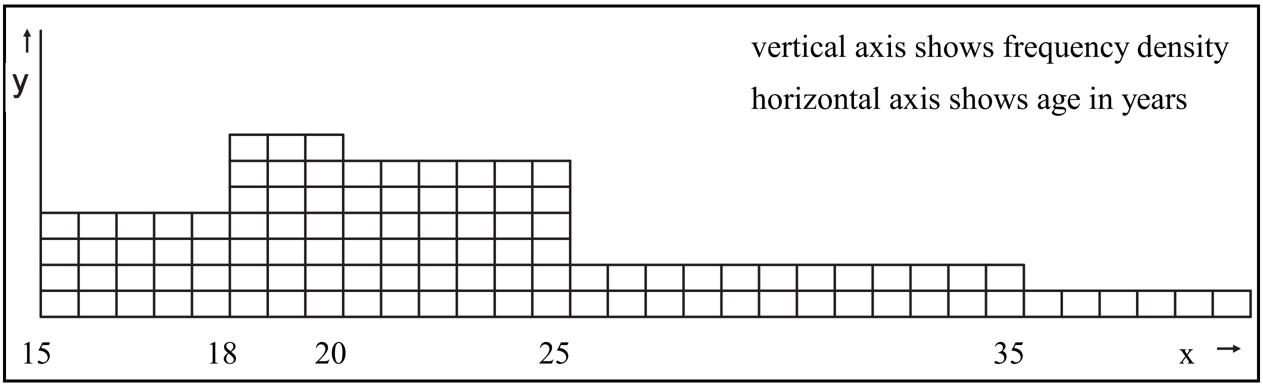
e) 's' is jointly proportional with the square of 'j' and inversely with the square root of 't'.

2) The rate at which an athletes heart beats is directly proportional to the time for which the athlete has been running. If an athlete has been running for two minutes the heart beats at a rate of 60 beats per minute. Find the rate at which the heart beats if the athlete has been running for five minutes.

3) The amount of water given to a plant is directly proportional to the square of its growth rate. When a plant is given 20ml of water it has a growth rate of 5mm per week. Find the rate of growth per week if the plant is given 35ml .

HISTOGRAMS 20

Below is an example of an histogram drawn to show the number of people who passed an aptitude test ranging from the ages of fifteen to fifty. In the range of fifteen to eighteen ten people passed the test.



It is obvious that it is very similar to a bar chart but the important exception is that the bars do not necessarily have to be equal in width, as it is the area of each bar which is important. By using the information given we can calculate the number of people represented by one square unit.

We are told that there are ten people in the age group of fifteen to eighteen and the area shown to represent the ten people is 20 square units.

10 / 20 = 0.5

Therefore each square unit represents 0.5 people.

We can therefore calculate the number of people in each age group which have passed by finding the area and multiplying it by 0.5. For example for the age group of 18 to 20 the area is:

Area = 3 X 7

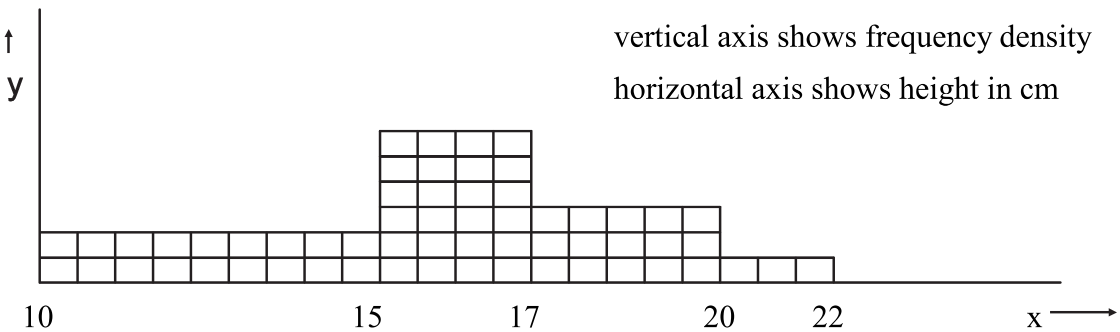
Area = 21

Number of people who passed = 21 X 0.5

between 18 and 20 = 10.5 = 11 (to the nearest whole number)

QUESTION

1) The histogram below shows the number of plants within the height ranges shown. Given that there are 3 plants within the height range of 10cm to 15cm and that they occupy an area of 18 square units; calculate the number of plants in all the other height ranges.



CUMULATIVE FREQUENCY CURVES 21

Cumulative frequency curves show the spread of data and allow you to calculate the median (middle value in a set of data) which is also known as the 50th percentile. Cumulative frequency curves also allow you to calculate the interquartile range which is the range of data between the upper quartile (75th percentile) and lower quartile (25th percentile).

Draw a typical cumulative frequency curve that you may be faced with in an exam.

You may be asked to find the median in which case you do the following:-

1) You look along the y-axis and divide the number at the top by two.

2) Whatever your answer is, you draw a horizontal line parallel to the x-axis until you reach the curve.

3) You then draw a vertical line which is parallel to the y-axis until you reach the x-axis. The value on the x-axis is the value of the median.

If asked to find the interquartile range you follow the steps given below:

1) You find the value of the upper quartile which is also known as the 75th percentile. You do this by multiplying the greatest value the curve reaches on the y-axis by 0.75. You then draw a horizontal line across from this value until you reach the curve and you then draw a vertical line until you reach the x-axis. The value on the x-axis is the value of the upper quartile.

2) You then find the value of the lower quartile range in the same way as above except you multiply by 0.25 instead of 0.75.

3) Once you have your upper and lower quartile values to find the interquartile range you take the lower quartile from the upper quartile:

Interquartile range = Upper quartile - Lower quartile

QUESTION

Calculate the value of the interquartile range for the cumulative frequency curve you have drawn above using the method provided.

ANSWER PAGE

Page 3:

2b) x = 342.6, x = 197.5

3) x = 63.43, x = 243.4, x = -116.6

4a) z = 0.866 (to 3 sig.fig.)

b) $x = 60^0$, $x = 120^0$

5) Y-limits are positive infinity and negative infinity and the graph repeats every 180^0.

Page 7:

1bi) 6/25 1bii) 9/25 1biii) 16/25

2b) 1 / 2197 2c) 469 / 2197

3a) 1/ 5525 3b) 3243 / 4225 3c) 1128 / 5525

Page 11:

1) $X^2 + 4X^3 - 12$

2) $3X^3 = 81$; $X = 3$

3) X = 1.41

4) $27X^2 + 30X - 8 = 0$

5a) 2b

5b) d / b

5c) s^3 / p^2

5d) $(d^2e^2 + ca^3) / ad$

5e)$[p - (r + p)^2] / (p + r)$

6a) $8rp (4r^2p - 2p + 3r) = y$

6b) $x (2x - 4y^3 + 2x^3) = z$

c) $4r (3r^2t + s^2 + r^3s) = 0$

Page 13:

1a) Is greater than

1b) Is less than or equal to

2a) $X < 2$

2b) $y > 1.5$

2c) $-6 \le x \le 6$

2d) $-1.83 \le x \le 1.83$

2e) $y < 81$

3) $-4 \le s \le 4$

Page 15:

1a) x = -1/ 3, x = -2

1b) x = -1, x = 1/ 2

1c) x = -8, x = 6

1d) x = -2, x = 3

1e) x = 3/ 5, x = 1

2a) x = 5, x = 1

2b) x = 1.92, x = -9.92

2c) x = 0.0990, x = -10.1

2d) Question 2a

3a) $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

3bi) x = -0.394, x = -7.61

3bii) x = 2.69 x = -0.186

Page 17:

1a) $x = 2\ {}^{7}/_{9}$ $y = -1\ {}^{1}/_{9}$

1b) x = 3 y = -1.5

1c) x = -1/ 3 y = 2/ 3

1d) x = -26 y = 96

Page 19:

1a) $y \ \alpha\ 1/\sqrt[3]{x}$

1b) $z \ \alpha\ r^4$

1c) $m \ \alpha\ o^2$

1d) $q \ \alpha\ 1/\sqrt{p}$

1e) $s \ \alpha\ j^2/\sqrt{t}$

2) R = 150

3) G = 6.12

Page 20:

Height range of 15 to 17 = 4 plants

Height range of 17 to 20 = 2.5 = 3 plants

Height range of 20 to 22 = 0.5 = 1 plant