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NOTE: All diagrams are deliberately approximate. This is to ensure that the answers cannot be easily calculated by simply measuring the lengths and angles from the diagrams, and also to test the students' ability to accurately draw diagrams from given information.

MATHEMATICS PACK 1

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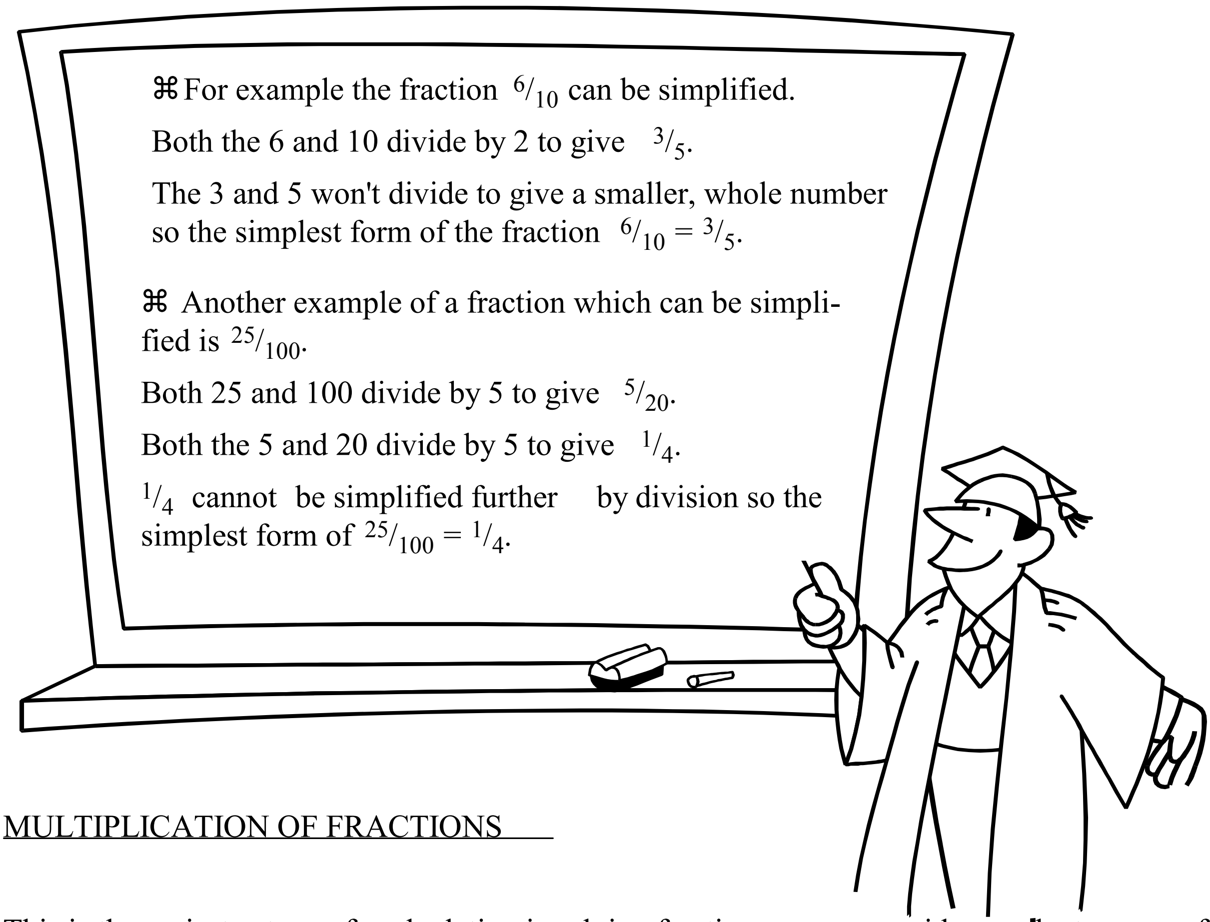
FRACTIONS 1

Fractions are rational numbers. There are four types of calculations you should be able to perform with fractions: multiplication, division, addition and subtraction. These calculations can be performed easily as long as you learn the rules associated with each type of calculation. Below is a description of these rules. However, it is important that you can firstly express a fraction in its simplest form.

SIMPLIFYING FRACTIONS

Most questions involving fractions will expect you to express the fraction obtained in its simplest form. Some fractions may already be in their simplest form, however others can be simplified by dividing the top and the bottom number by the same number until they won't divide any more.



MULTIPLICATION OF FRACTIONS

This is the easiest type of calculation involving fractions as you consider the top row of the fractions and the bottom row of the fractions separately. The rule is to simply multiply the numbers on the top row together and put the answer over what you get when you multiply the numbers on the bottom row together.

⌘Worked Example 1

Calculate $^{3}/_{7} \cdot {}^{8}/_{13}$

STEP 1: Consider the top row and bottom row of the fractions separately.

i.e. $3 \cdot 8 = 24$

$7 \cdot 13 = 91$

STEP 2: Put the value obtained from multiplying the numbers on the top row over the value obtained from multiplying the numbers on the bottom row so that you get $^{24}/_{91}$

Therefore $^{3}/_{7} \cdot {}^{8}/_{13} = {}^{24}/_{91}$

⌘Worked Example 2

Calculate $^{9}/_{11} \cdot {}^{15}/_{17} \cdot {}^{1}/_{3}$

STEP 1: Consider the top row and bottom row of the fractions separately.

$9 \cdot 15 \cdot 1 = 135$

$11 \cdot 17 \cdot 3 = 561$

STEP 2: Put the value obtained from multiplying the numbers on the top row over the value obtained from multiplying the numbers on the bottom row so that you get $^{135}/_{561}$

Therefore $^{9}/_{11} \cdot {}^{15}/_{17} \cdot {}^{1}/_{3} = {}^{135}/_{561}$

$^{135}/_{561}$ can be simplified to give $^{45}/_{187}$ as both 135 and 561 divide by 3. They won't divide any further to give a smaller, whole number.

Therefore $^{9}/_{11} \cdot {}^{15}/_{17} \cdot {}^{1}/_{3} = {}^{45}/_{187}$

The above method for multiplying fractions is used to solve probability questions, which is dealt with in the next section of this book.

DIVISION OF FRACTIONS

To divide a fraction by another fraction you should use the following steps:

STEP 1 Turn the second fraction upside down and replace the division sign (|) by the multiplication sign (·).

STEP 2 Use the multiplication rule.

⌘Worked Example

Find the value of $^{4}/_{7} \mid {}^{11}/_{9}$

STEP 1 Turn the second fraction upside down and replace the | sign by the ◦ sign so that the calculation reads:

$^{4}/_{7} \cdot {}^{9}/_{11}$

STEP 2 Use the multiplication rule.

Put the value obtained from multiplying the numbers on the top row over the value obtained from multiplying the numbers on the bottom row.

Therefore $^{4}/_{7} \mid {}^{11}/_{9} = {}^{36}/_{77}$

ADDITION OF FRACTIONS

Use the following simple steps to add fractions together.

STEP 1 Find a common denominator (this means make the numbers on the bottom of each fraction the same). Then adjust the values on the top of each fraction accordingly.

STEP 2 Add the numbers on the top row of each fraction together and put your answer over the common denominator.

STEP 3 Simplify the fraction, if necessary, using the method described earlier.

⌘Worked Example

Calculate $^5/_{11} + ^3/_4$. Express your answer in its simplest form.

STEP 1 Find a common denominator.

This can be done by multiplying the numbers on the bottom row of each fraction together. In this case you need to multiply 4 and 11 together.

$4 \cdot 11 = 44$

Now the numbers on the top of each fraction must be changed accordingly. This is done by multiplying the top number of the first fraction by the bottom number of the second fraction.

i.e. $5 \cdot 4 = 20$

Then multiply the top number of the second fraction by the bottom number of the first fraction.

i.e. $3 \cdot 11 = 33$

As a result of the above steps the equation should read:

$$\frac{(5 \cdot 4)}{(11 \cdot 4)} + \frac{(3 \cdot 11)}{(11 \cdot 4)}$$

This can be simplified to give $^{20}/_{44} + ^{33}/_{44}$

STEP 2 Add the numbers on the top row of each fraction together.

$20 + 33 = 53$

Put your answer over the common denominator.

Therefore $^{20}/_{44} + ^{33}/_{44} = ^{53}/_{44}$

STEP 3 Simplify.

$^{53}/_{44}$ is a top heavy fraction as the number of the top of the fraction is larger than the number on the bottom of the fraction. It can therefore be simplified to give $1\ ^9/_{44}$.

As the fraction is top heavy you first write down the number 1, as 44 will divide into 53 once and then give a remainder. To find the value of the remainder you perform the following calculation: $53 - 44 = 9$ Therefore the remaining fraction is $^9/_{44}$.

$^{53}/_{44} = 1\ ^9/_{44}$

Therefore $^5/_{11} + ^3/_4 = 1\ ^9/_{44}$

SUBTRACTION OF FRACTIONS

To subtract one fraction from another the method is very similar to that used to add fractions together. *Notice it is only the second step that changes where as the other steps remain the same as in addition of fractions.*

STEP 1 Find a common denominator (this means make the numbers on the bottom of each fraction the same). Then adjust the values on the top of each fraction accordingly.

STEP 2 Subtract the number on the top row of one fraction from the number on the top row of the other fraction and put your answer over the common denominator.

STEP 3 Simplify the fraction, if necessary, using the method described earlier.

⌘ Worked Example

Calculate $^3/_4$ - $^1/_3$, expressing your answer in its simplest form.

STEP 1 Find a common denominator.

This can be done by multiplying the numbers on the bottom row of each fraction together. In this case you need to multiply 4 and 3 together.

$4 \cdot 3 = 12$

Now the numbers on the top of each fraction must be changed accordingly. This is done by multiplying the top number of the first fraction by the bottom number of the second fraction.

i.e. $3 \cdot 3 = 9$

Then multiply the top number of the second fraction by the bottom number of the first fraction.

i.e. $1 \cdot 4 = 4$

As a result of the above steps the equation should read:

$$\frac{(3 \cdot 3)}{(4 \cdot 3)} - \frac{(1 \cdot 4)}{(4 \cdot 3)}$$

This can be simplified to give $^9/_{12}$ - $^4/_{12}$

STEP 2 Subtract the number on the top row of the second fraction from the number on the top row of the first fraction.

$9 - 4 = 5$

Put your answer over the common denominator.

Therefore $^9/_{12} - {}^4/_{12} = {}^5/_{12}$

STEP 3 Simplify

The answer $^5/_{12}$ cannot be simplified further as the 5 and 12 do not divide to give a smaller, whole number.

Therefore $^3/_4 - {}^1/_3 = {}^5/_{12}$

QUESTIONS ON FRACTIONS 5

EXERCISE 1A Calculate the following giving your answer in its simplest form.

1) $^{3}/_{5} \cdot {}^{1}/_{5}$
2) $^{12}/_{13} \cdot {}^{7}/_{8}$
3) $^{1}/_{2} \cdot {}^{1}/_{4} \cdot {}^{1}/_{5}$
4) $^{7}/_{11} \cdot {}^{2}/_{3}$
5) $^{15}/_{23} \cdot {}^{9}/_{10}$
6) $^{19}/_{7} \cdot {}^{5}/_{7}$
7) $^{3}/_{100} \cdot {}^{1}/_{3} \cdot 10$

<u>EXERCISE 1B</u>

Calculate the following, showing all your working and give your answer in its simplest form.

1) $^{5}/_{12} \mid {}^{8}/_{9}$
2) $^{7}/_{13} \mid {}^{7}/_{11}$
3) $^{5}/_{6} \mid {}^{1}/_{23}$
4) $^{6}/_{49} \mid {}^{7}/_{3}$
5) $^{5}/_{7} \mid {}^{3}/_{8}$
6) $^{9}/_{14} \mid {}^{4}/_{5}$
7) $^{3}/_{20} \mid {}^{3}/_{4}$

<u>EXERCISE 1C</u>

Calculate the following, showing all your working and giving your answer in its simplest form.

1) $^{2}/_{5} + {}^{1}/_{6}$
2) $^{2}/_{5} + {}^{13}/_{15}$
3) $^{1}/_{16} + {}^{6}/_{7}$
4) $^{1}/_{2} + {}^{3}/_{14}$

<u>EXERCISE 1D</u>

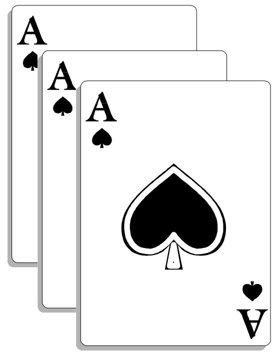
Calculate the following, showing all your working and giving your answer in its simplest form.

1) $^{3}/_{10} - {}^{3}/_{16}$
2) $^{17}/_{2} - {}^{6}/_{11}$
3) $^{99}/_{100} - {}^{1}/_{4}$
4) $^{1}/_{2} - {}^{3}/_{14}$

<u>EXERCISE 1E</u> Calculate the following, showing all your working and giving your answer in its simplest form. *Hint - You should work out the brackets first.*

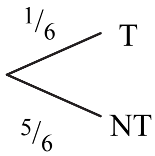
1) $(^{9}/_{11} - {}^{2}/_{3}) \mid {}^{1}/_{3}$
2) $(^{2}/_{9} + {}^{1}/_{12}) \cdot {}^{8}/_{3}$

PROBABILITY 6



Probability is the likelihood that an event will or will not take place. For example a question may involve finding the probability that a person will pick four aces from a normal pack of cards.

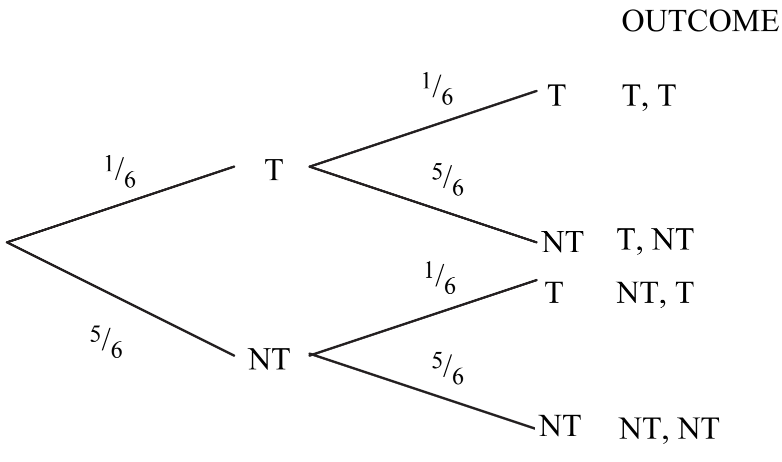
When faced with a question involving calculating the probability that something will or will not take place it is a good idea to draw a tree diagram using the information provided in the question. Below is a tree diagram which represents the probability of obtaining a three when a fair die is thrown once:



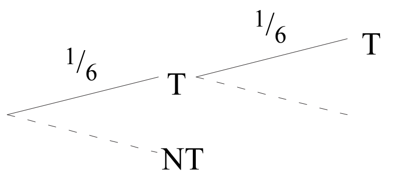
T = THREE

NT = NOT A THREE

The tree diagram has two branches one which shows the probability of obtaining a three, the other shows the probability of not obtaining a three. There are six possible outcomes when a six sided die is thrown and only one of the six sides of the die shows the number three. Therefore the probability of obtaining a three is one in six or $^1/_6$. The other five sides of the die do not show the number three so the probability of not obtaining a three is five in six or $^5/_6$. You can check this tree diagram is correct by finding the sum of the probabilities extending from one point; if the diagram is correct the sum of the probabilities should be one, ($^5/_6$ + $^1/_6$ = 1). Below is another three diagram which shows the probability of obtaining a three when a fair die is thrown twice.



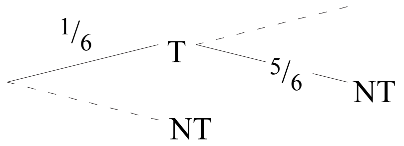
⌘To calculate the probability of obtaining a three on both throws you follow along the branches which lead to two threes being obtained (these have been highlighted for you). You multiply the probabilities together which are associated with each of the desired branches to get your final answer. For the above diagram you would perform the following calculation to find the probability of obtaining a three on both throws.



Probability of 2 threes = $^1/_6 \cdot {}^1/_6$

$= {}^1/_{36}$

⌘To calculate the probability of obtaining a three on either the first throw or the second throw the calculation is slightly different. First you find the probability of obtaining a three followed by the probability of not obtaining a three. You do this as above by multiplying the probabilities together which are associated with each of the branches required.

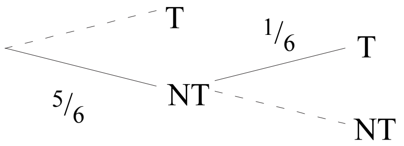


Probability of obtaining a three then not a three = $^1/_6 \cdot {}^5/_6$

$= {}^5/_{36}$

You then find the probability of not obtaining a three followed by the probability of obtaining a three.

Again you multiply together the probabilities associated with the branches which lead to this outcome.



Probability of not obtaining a three on the first throw = $^5/_6 \cdot {}^1/_6 = {}^5/_{36}$

followed by a three on the second throw.

You can now find the probability of obtaining a three when a die is thrown twice by adding together the probabilities of the two possible outcomes. The two possible outcomes and their associated probabilities are shown below.

Probability of a three then not a three = $^5/_{36}$

Probability of not obtaining a three then a three = $^5/_{36}$

Therefore probability of obtaining a three in two throws = $^5/_{36} + {}^5/_{36}$

$= {}^{10}/_{36}$

$= {}^5/_{18}$

⌘ Sometimes a question may ask you to find the probability of obtaining 'at least' one three when the die is thrown twice. This means there are three possible outcomes:

1) Obtaining a three followed by another three.

2) A three on the first throw followed by not a three on the second throw.

3) Not a three on the first throw but a three on the second throw of the die.

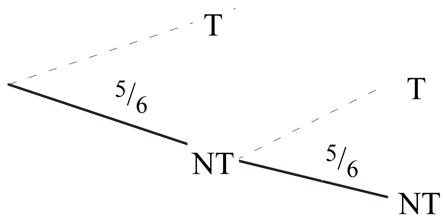
An easier way of interpreting 'at least' questions is to find the probability of the other possible outcome/outcomes and subtract this probability from the number 1.

i.e. probability of obtaining at least one = 1 - probability of other outcomes.

The only other possible outcome is to obtain no threes: not a three followed by not a three.

Probability of no threes on the two throws = $^5/_6 \cdot {}^5/_6$

$= {}^{25}/_{36}$



Therefore the probability of obtaining at least one three = 1 - $^{25}/_{36}$

$= {}^{11}/_{36}$

You should always check your tree diagram is correct by finding the sum of the probabilities on branches extending from one point; if the diagram is correct the sum of the probabilities should be one.

⌘ To calculate the final answer you multiply together the probabilities along the desired branches. (As shown)

⌘ When the question involves finding the probability of 'either' one outcome or another you calculate the probabilities of each outcome separately and you then add together these probabilities. (As shown above)

⌘ When answering an 'at least' question it is usually easier to find the probability of the other possible outcomes and subtract these from one.

WORKED EXAMPLE 1

Question

Samantha has a bag of 17 marbles containing red marbles and blue marbles. If she picks a marble out of the bag the probability/chance of her picking a red marble is $^{7}/_{17}$.

1) What is the probability of Samantha picking a blue marble from the bag.

2) If Samantha picks a marble from the bag and then replaces it before picking another marble what is the probability that she picks

a) Two red marble

b) Two blue marbles

c) A red marble followed by a blue marble

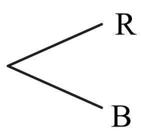
d) A blue marble followed by a red marble

e) A blue marble on either the first or second pick

f) At least one red marble.

Answer

Before attempting the question remember to draw a tree diagram displaying the information given in the question.



R = RED MARBLE

B = BLUE MARBLE

1) To find the probability of picking a blue marble from the bag you should remember that the sum of the probabilities extending from one point equal one. We already know that the probability of picking a red marble is $^{7}/_{17}$ so to find the probability of picking a blue marble you perform the following calculation:

Sum of probabilities = 1

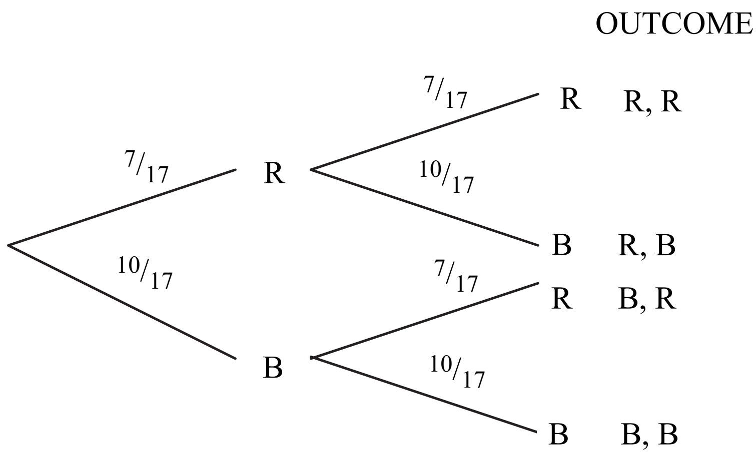
Probability of picking a red marble + Probability of picking a blue marble = 1

$^{7}/_{17}$ + Probability of picking a blue marble = 1

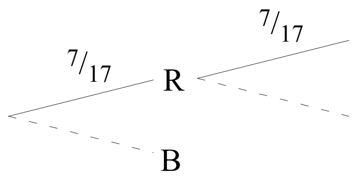
Probability of picking a blue marble = 1 - $^{7}/_{17}$

Probability of picking a blue marble = $^{10}/_{17}$

2) Before attempting this question you need to draw another tree diagram showing the possible outcomes when Samantha picks two marbles with replacement.



2a) To find the probability of obtaining two red marbles you should remember that you need to multiply together the probabilities along the branches which lead to the desired outcome. The part of the tree diagram which leads to the outcome of two red marbles being picked is shown above.

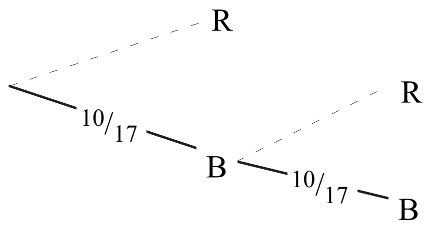


The calculation you should perform is below:

Probability of picking two red marbles = $^{7}/_{17} \cdot {^{7}/_{17}}$

$= {^{49}/_{289}}$

2b) To find the probability of obtaining two blue marbles you should again remember that you need to multiply together the probabilities along the branches which lead to the desired outcome. The part of the tree diagram which leads to the outcome of two blue marbles being picked is shown below.

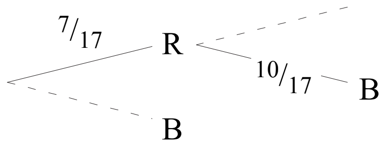


The calculation you should perform is given below:

Probability of picking two blue marbles = $^{10}/_{17} \cdot {^{10}/_{17}}$

$= {^{100}/_{289}}$

2c) To find the probability of picking a red marble followed by a blue marble you should again remember that you need to multiply together the probabilities along the branches which lead to the desired outcome. The part of the tree diagram which leads to the outcome of a red marble being picked and then a blue marble being picked is shown below.

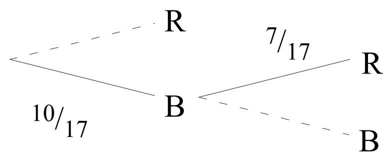


The calculation you should perform is given below:

Probability of picking a red marble followed by a blue marble $= {^{7}/_{17}} \cdot {^{10}/_{17}}$

$= {^{70}/_{289}}$

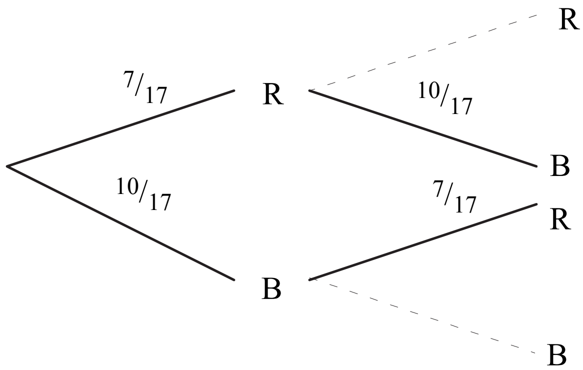
2d) This calculation is the same as that for 2c as the probabilities of picking a blue marble and a red marble remain the same. The part of the tree diagram used is shown below.



Probability of picking a blue marble followed by a red marble = $^{10}/_{17} \cdot {}^{7}/_{17}$

$= {}^{70}/_{289}$

2e) To find the probability of obtaining a blue marble on either the first pick or the second pick you should notice the key word 'either' which should remind you to calculate the probabilities of each possible outcome separately and then add these probabilities together. The tree diagram shows that there are two routes which lead to a blue marble being picked:



The calculation to be performed is:

Probability of obtaining a blue on either the first or the second pick.	=	Probability of picking a red marble followed by a blue marble	+	Probability of picking a blue marble followed by a red marble

In question (2c) we have already found the probability of picking a red marble followed by a blue marble is $^{70}/_{289}$. In question (2d) we have found the probability of picking a blue marble followed by a red marble is $^{70}/_{289}$. We now substitute these probabilities into the above equation.

Probability of obtaining a blue marble on either the first or second pick $= {}^{70}/_{289} + {}^{70}/_{289}$

$= {}^{140}/_{289}$

2f) Find the probability of picking at least one red marble on the either of the two attempts.

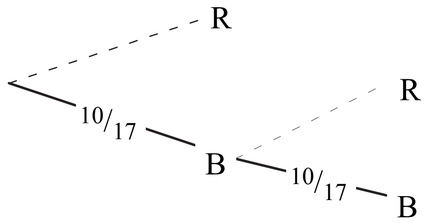
To find the probability of picking at least one red marble you have to find the probability of picking:

a red marble followed by a blue marble,

a blue marble followed by a red marble,

and the probability of picking two red marbles.

You then have to add the probabilities for each of the above together to get your answer. This method is very lengthy so you should remember that with 'at least' questions it is easier to find the probabilities of the other possible outcomes and subtract these from one. The only other possible outcome is to pick no red marbles on the two attempts; this can be seen easily on the first tree diagram. To calculate the probability of picking no red marbles you need to multiply together the probabilities along the branches which lead to this outcome. The part of the tree diagram you should use is shown below:



Probability of picking no red marbles = Probability of picking two blue marbles (found in 2b)

Probability of picking no red marbles = $^{100}/_{289}$

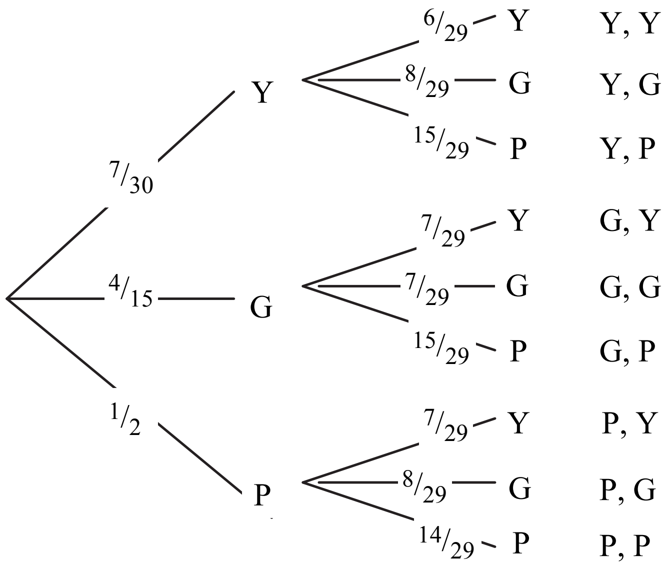
Now you can find the probability of picking at least one red marble by subtracting the probability of picking no red marbles from 1.

Probability of picking at least one red marble = 1 - $^{100}/_{289}$

= $^{189}/_{289}$

WORKED EXAMPLE 2

Jimmy has a packet of 30 jelly beans. Seven of the jelly beans are yellow, eight of the jelly beans are green and the rest are purple. Draw a tree diagram showing the above information, along with the probabilities of picking each of the three different coloured jelly beans, if Jimmy picks one jelly bean and eats it and then picks another jelly bean.



⌘ When Jimmy picks the first jelly bean the probability of it being

Yellow is $^{7}/_{30}$ as there are thirty jelly beans in total and we are told seven of them are yellow,

green is $^{8}/_{30}$ as there are thirty jelly beans in total and we are told eight of them are green,

purple is $^{15}/_{30}$ as there are thirty jelly beans in total and we are told fifteen of 30 are purple.

Probability of yellow is $^{7}/_{30}$ Probability of green is $^{8}/_{30} = {}^{4}/_{15}$ Probability of purple is $^{15}/_{30} = {}^{1}/_{2}$

Once Jimmy has eaten the first jelly bean there are no longer thirty jelly beans left in the packet, there are now twenty nine jelly beans. As a result of this the probabilities of picking a particular colour also change. Below are some examples, for the second pick, explaining why the tree diagram above has been given the probabilities for particular colours.

⌘ When the first jelly bean is yellow the probability of the second being:

yellow is $^{6}/_{29}$. This is because there are now twenty nine jelly beans in total, instead of thirty, as one has been eaten. There were originally seven yellow jelly beans and as the jelly bean eaten was yellow there are only six left in the packet.

green is $^{8}/_{29}$. This is because there are now twenty nine jelly beans in total, instead of thirty, as one has been eaten. There were originally eight green jelly beans and as the jelly bean eaten was yellow there are still eight green jelly beans left in the packet.

purple is $^{15}/_{29}$. This is because there are now twenty nine jelly beans in total, instead of thirty, as one has been eaten. There were originally fifteen purple jelly beans and as the jelly bean eaten was yellow there are still fifteen purple jelly beans left in the packet.

Probability of yellow = $^{6}/_{29}$ Probability of green = $^{8}/_{29}$ Probability of purple = $^{15}/_{29}$

QUESTIONS ON PROBABILITY

EXERCISE 2A

1) A boy buys a pack of 52 playing cards and takes a card at random.

Find the probability of the boy picking a card which is a queen.

2) The boy then replaces the first card and takes another card from the pack.

Find the probability of the boy picking a card which is an ace.

3) A girl takes a card from the pack of 52 playing cards and the card she takes is an ace. The girl does not put the card back.

Find the probability that the second card she takes from the pack is also an ace.

EXERCISE 2B

A box contains twenty felt tip pens. Four of the felt tip pens are orange, six of the felt tip pens are pink and ten of the felt tip pens are brown. A woman takes a felt tip pen from the box. A man then takes a felt tip pen from the box.

1) Draw a tree diagram representing the above information along with the associated probabilities of each branch.

Use your tree diagram to help you answer the following questions.

2) Find the probability of a man taking a brown felt tip pen and the woman taking a brown felt tip pen.

3) Find the probability of an orange felt tip pen being taken followed by a pink felt tip pen.

4) Find the probability of no orange felt tip pens being taken.

5) Find the probability that at least one orange felt tip pen being taken.

6) Find the probability of a pink pen being taken either by the man or the woman.

7) Find the probability of two pink felt tip pens being taken.

8) Find the probability that at least one pink felt tip pen is taken.

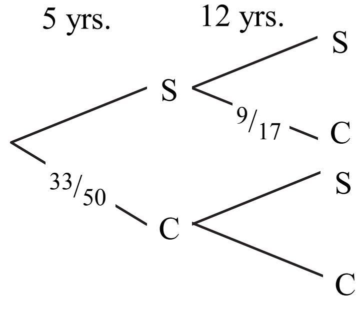
EXERCISE 2C

1) A scientist wants to do an experiment. The scientist asks fifty 5 year old girls to draw a square or a circle. Thirty three of the 5 year old girls draw a circle.

How many of the girls drew a square.

2) The scientist then ask fifty 12 year old girls to draw a square (S) or a circle (C). Eighteen of the 12 year old girls draw a circle.

Fill in the missing probabilities in the tree diagram below.



3) Find the probability that no squares are drawn by the 5 year old girls or the 12 year old girls.

4) Find the probability of a 5 year old girl drawing a circle and a 12 year old girl then drawing a square.

FACTORS, MULTIPLES & PRIME FACTORS 15

⌘ A FACTOR is a number which divides into a given number exactly.

⌘ When a number is multiplied by another number it gives the MULTIPLE.

⌘ The PRIME FACTORS of a number are the smallest prime numbers that divide into the given number.

FACTORS

If a question asks you to find the factors of a number it is asking you to find all the numbers which divide into it. As long as you follow the method given below finding the factors is a simple process.

Method:

1) Divide the number given by 1. All numbers divide by one exactly so the number one is a factor.

2) Then divide the number by 2. If the number divides by two exactly, i.e. there are no remainders, the number two and the number produced on division are both factors. If the number given does not divide by two exactly it is not a factor.

3) Divide the number given by 3, 4, 5 and so on, writing down the factors that you find.

4) Once the numbers start to repeat you know you have found all the factors.

WORKED EXAMPLE 1

Find the factors of 28.

Method:

1) $28 \mid 1 = 28$

Therefore 28 and the number 1 are both factors of 28.

2) $28 \mid 2 = 14$

Therefore 2 and 14 are factors of 28.

3) $28 \mid 3 = 9\ ^{1}/_{3}$

Therefore 3 is not a factor of 28 as three does not divide into twenty eight exactly.

4) $28 \mid 4 = 7$

Therefore 4 and 7 are factors of 28.

5) $28 \mid 5 = 5.6$

$28 \mid 6 = 4\ ^{2}/_{3}$

Therefore 5 and 6 are not factors of 28 as five and six do not divide into 28 exactly.

6) $28 \mid 7 = 4$

These numbers appeared in step four so we must have found all the factors of 28.

Factors of 28: 1, 2, 4, 7, 14, 28.

WORKED EXAMPLE 2

Find the factors of 6.

Method:

1) 6 | 1 = 6

Therefore 6 and 1 are both factors of 6 as they divide into six exactly.

2) 6 | 2 = 3

Therefore 2 and 3 are both factors of 6 as they divide into six exactly.

3) 6 | 3 = 2

These numbers appeared in step two so we have found all the factors of six.

Factors of 6: 1, 2, 3, 6.

MULTIPLES

If a question asks you to find the first four multiples of a number it is asking you to multiply the number by one, then two, three and four. If we say the given number is 'x' we use the following method to find the first four multiples.

1) First multiply 'x' by 1 to find the first multiple.

$x \cdot 1 = x$

2) Now multiply 'x' by 2 to find the second multiple. Then multiply 'x' by 3 and then 4 to find the third and fourth multiples.

WORKED EXAMPLE 1

Find the first six multiples of three.

$3 \cdot 1 = 3$

$3 \cdot 2 = 6$

$3 \cdot 3 = 9$

$3 \cdot 4 = 12$

$3 \cdot 5 = 15$

$3 \cdot 6 = 18$

The first six multiples of 3 are: 3, 6, 9, 12, 15, 18.

WORKED EXAMPLE 2

Find the first five multiples of eleven.

$11 \cdot 1 = 11$

$11 \cdot 2 = 22$

$11 \cdot 3 = 33$

$11 \cdot 4 = 44$

$11 \cdot 5 = 55$

The first five multiples of 11 are: 11, 22, 33, 44, 55.

PRIME FACTORS

A prime number is one which only divides by itself and one. When a question asks you express a number as a product of prime factors it is asking you to find the smallest prime numbers that divide into the given number.

Method :

Let us say the given number is 'Z.'

1) Find the smallest prime number that divides into Z. First divide Z by the number 2 which is the smallest prime number; if Z divides exactly by 2 to give another number you go on to the next step. If Z does not divide by 2 exactly you then see if it will divide by the next smallest prime number which is 3; if Z divides by 3 you go onto the next step. If Z does not divide by 3 exactly you divide Z by the next smallest prime number etc. until you find a prime number which does divide into Z.

You now have a different number as a result of the division in step one. We can call this number Y.

2) You then find the smallest prime number that divides into Y, using the same method as described in step one, this will give you another number.

3) Repeat step two until you arrive at a number which when divided by its smallest prime number gives you the number 1. When you arrive at the number one you have found the prime factors.

WORKED EXAMPLE 1

Express fifteen as a product of prime factors.

1) Find the smallest prime number that divides into fifteen.

$15 \mid 2 = 7.5$

As fifteen does not divide by two exactly you know it is not the smallest prime factor.

2) $15 \mid 3 = 5$

Fifteen does divide by three exactly so you know three is the smallest prime factor.

3) You now need to find the smallest prime number that divides into five.

$5 \mid 2 = 2.5$

$5 \mid 3 = 1\ ^{2}/_{3}$

As five does not divide by two and three they are not prime factors of five.

3) $5 \mid 5 = 1$

As we have arrived at the number one we know we have found all the prime factors of fifteen. We express our answer as follows:

$15 = 3 \circ 5$

WORKED EXAMPLE 2

Express twenty as a product of prime factors.

1) Find the smallest prime factor of twenty:

20 | 2 = 10

As two divides into twenty exactly it is the smallest prime factor.

2) Find the smallest prime factor of ten:

10 | 2 = 5

Two divides into ten exactly so you it is the smallest prime factor of ten.

3) Find the smallest prime number that divides into five exactly.

5 | 2 = 2.5

5 | 3 = 1 $^{2}/_{3}$

5 | 5 = 1

Five divides by five to give one so we have found all the prime factors of twenty. We express our answer as follows.

$20 = 2 \circ 2 \circ 5$

LOWEST COMMON MULTIPLE (LCM)

Some questions may ask you to find the lowest common multiple (LCM) of numbers. To find the lowest common multiple of the numbers given you simply multiply them until a common multiple for the numbers is found.

For example when asked to find the lowest common multiple for the numbers 3 and 7 you write down some multiples of three and some multiples of seven.

Multiples of three: 3, 6, 9, 12, 15.

Multiples of seven: 7, 14, 21, 28, 35.

The above shows that a common multiple has not yet been found so you continue to write down multiples of three.

Multiples of three: 18, 21, 24, 27, 30.

From the above it is now possible to see that the number 21 is a common multiple for both 3 and 7. There are other common multiples of 3 and 7 but they are greater than 21; these include the numbers 42 and 63.

In some cases it may be necessary to write down further multiples of seven if a common multiple can not be found.

Therefore the lowest common multiple (LCM) of 3 and 7 = 21.

HIGHEST COMMON FACTOR (HCF)

When asked to find the highest common factor of some given numbers you should do the following:

1) Write down the factors of each number given. (These are numbers which divide exactly into the given number).

2) Compare the factors for each number and identify any common factors.

3) From the common factors identified write down the one which is the highest and that is your highest common factor.

For example if asked to find the highest common factor for the numbers 12 and 38 you would do the following:

Write down the factors of twelve:

Factors of 12 are: 1, 2, 3, 4, 6, 12.

Factors of 38 are: 1, 2, 19, 38.

From the above lists the highest common factor (HCF) = 2

QUESTIONS ON FACTORS, MULTIPLES & PRIME FACTORS

EXERCISE 3A

1) Find the factors of:

a) 30
b) 14
c) 4
d) 25
e) 36
f) 9
g) 18
h) 10
i) 50

EXERCISE 3B

1) Write down the first five multiples of

a) 4
b) 5
c) 12
d) 10
e) 2
f) 9

2) Write down the first seven multiples of

a) 6
b) 8
c) 14
d) 7
e) 13
f) 100

EXERCISE 3C

1) Express the following as a product of prime factors:

a) 14
b) 22
c) 36
d) 50
e) 33
f) 42
g) 45
h) 60
i) 5

EXERCISE 3D

1) Find the lowest common multiple of

a) 6 and 7
b) 12 and 18
c) 4 and 11
d) 13 and 4
e) 15 and 6
f) 3 and 6

2) Find the highest common factor of

a) 26 and 52
b) 44 and 33
c) 51 and 12
d) 24 and 36
e) 5 and 16
f) 32 and 28

ANGLES OF REGULAR POLYGONS 21

Questions involving finding angles of regular polygons usually ask you to find the interior or exterior angle of the regular polygon.

A polygon is a shape with many sides.

A regular polygon is a shape in which all the sides and angles are equal.

An interior angle is an angle on the inside of the regular polygon.

An exterior angle is an angle on the outside of the regular polygon.

Below are some facts which make answering questions involving regular polygons easy.

⌘Equation 1

Exterior angle = 360 | number of sides

⌘Equation 2

Interior angle = 180 - exterior angle

The above facts can be used to answer questions which involve you working backwards.

For example a question may ask you to find the number of sides of the regular polygon given the exterior angle. To answer this you would rearrange the first fact so that it reads:

⌘Number of sides = 360 | exterior angle

Another question may ask you to find the number of sides a regular polygon has given the interior angle. In this case you would first find the value of the exterior angle by rearranging the second fact so that it reads:

⌘Exterior angle = 180 - Interior angle

You would then find the number of sides by rearranging the first fact so that it reads:

⌘Number of sides = 360 | Exterior angle

SHAPE	NUMBER OF SIDES
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10

<u>WORKED EXAMPLE 1</u>

Find the value of the interior angle of a regular octagon.

To answer this question you first need to know that an octagon has eight sides.

You should first find the value of the exterior angle:

Exterior angle = 360 | number of sides

Exterior angle = 360 | 8

Exterior angle = 45^0

Interior angle = 180 - Exterior angle

Interior angle = 180 - 45

Interior angle = 135^0

<u>WORKED EXAMPLE 2</u>

If a regular polygon has an interior angle of 140^0 how many sides does the regular polygon have.

To answer this question you first rearrange equation 2 so that it reads:

Exterior angle = 180 - Interior angle.

Exterior angle = 180 - 140

Exterior angle = 40^0

You then rearrange equation one so that it reads:

Number of sides = 360 | Exterior angle

Number of sides = 360 | 40

Number of sides = 9 (Therefore the regular polygon is a nonagon.)

QUESTIONS ON ANGLES OF REGULAR POLYGONS

<u>EXERCISE 4A</u>

1) Find the exterior angle of a heptagon, giving your answer to one decimal place.

2) Find the exterior angle of a triangle.

3) A decagon has an exterior angle of 36^0, find the value of the interior angle of a decagon.

4) A regular polygon has an interior angle of 60^0, find the value of the exterior angle.

<u>EXERCISE 4B</u>

1) A regular polygon has eleven sides, find the value of

a) the exterior angle

b) the interior angle

giving your answers to one decimal place.

2) If a regular polygon has an exterior angle of 30^0 how many sides does it have.

3) If a regular polygon has an exterior angle of 72^0 how many sides does it have.

<u>EXERCISE 4C</u>

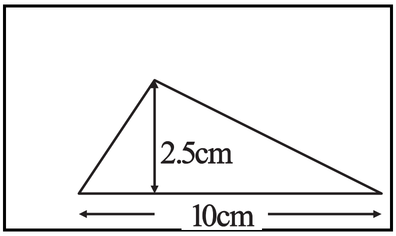
1) A regular polygon has interior angles of 120^0 find

a) the value of the exterior angle

b) the number of sides the regular polygon has and name the polygon.

2) If a regular polygon has interior angles of 154.29^0 (correct to two decimal places), how many sides does the regular polygon have.

AREAS AND VOLUMES

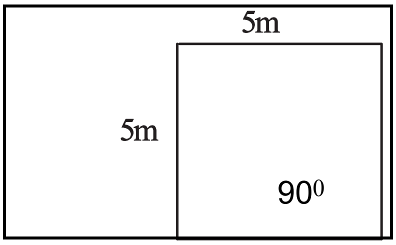


Area Of A Triangle = 1/2 × Base × Height

WORKED EXAMPLE FOR DIAGRAM

Area Of Triangle = 1/2 × 10 × 2.5

Area Of Triangle = 12.5 cm^2



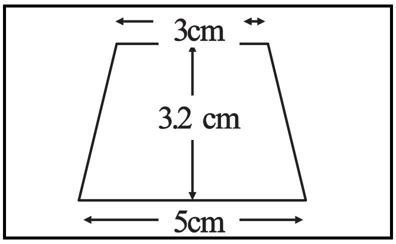
Area Of A Square = Base × Height

NB: Base = Height

WORKED EXAMPLE FOR DIAGRAM

Area Of Square = 5 × 5

Area Of Square = 25m^2

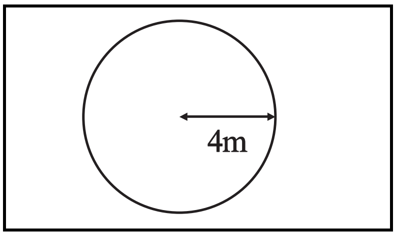


Area Of Trapezium = Vertical Height × Average Of Parallel sides

WORKED EXAMPLE FOR DIAGRAM

Area Of Trapezium = 3.2 × ((5 + 3)÷ 2)

Area Of Trapezium = 12.8 cm^2

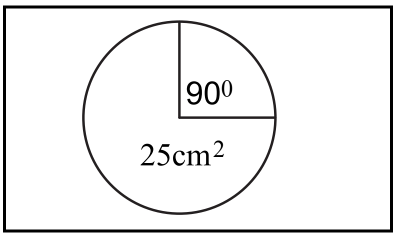


Area Of Circle = π × $radius^2$

WORKED EXAMPLE FOR DIAGRAM

Area Of Circle = $\pi \times 4 \times 4$

Area Of Circle = 50.27 m^2 (to 2 d.p.)

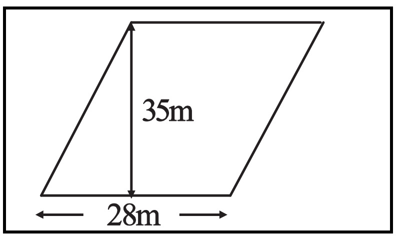


Area Of Sector = (Angle/ 360) × Area Of Circle

WORKED EXAMPLE FOR DIAGRAM

Area Of Sector = (90÷ 360) × 25

Area Of Sector = 6.25 cm^2

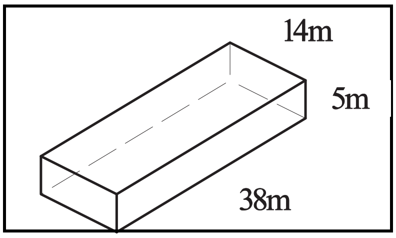


Area Of A Parallelogram = Base × Height

WORKED EXAMPLE FOR DIAGRAM

Area Of Parallelogram = 28 × 35

Area Of Parallelogram = 980m^2

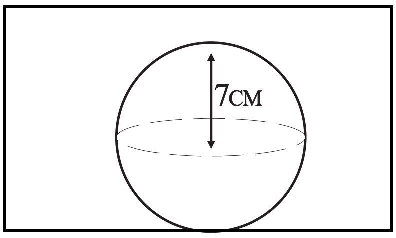


Volume Of A Prism = Area Cross Section × length

WORKED EXAMPLE FOR DIAGRAM

Volume Of Prism = (14 × 5) × 38 = 2660 m

Volume Of Prism = $2660m^3$

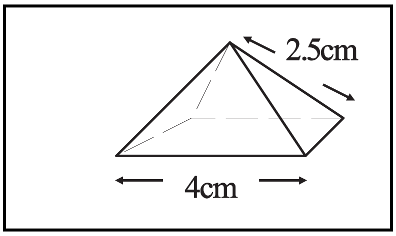


Volume Of A Sphere = 4/3 × π × $radius^3$

WORKED EXAMPLE FOR DIAGRAM

Volume Of Sphere = 4/3 × π × 7^3

Volume Of Sphere = $1437cm^3$ (to 4 sig.fig.)

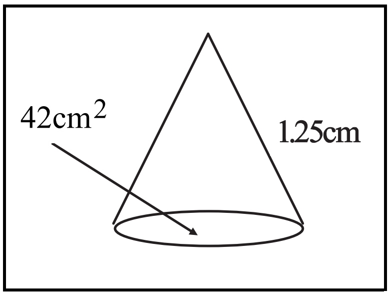


Volume Of A Pyramid = 1/3 × Area Of Base × Height

WORKED EXAMPLE FOR DIAGRAM

Volume Of Pyramid = 1/3 × 16 × 2.5

Volume Of Pyramid = $13.33cm^3$ (to 2 d.p.)



Volume Of A Cone = 1/3 × Area Of Circular Base × Height

WORKED EXAMPLE FOR DIAGRAM

Volume Of Cone = 1/3 × 42 × 1.25

Volume Of Cone = 17.5 cm^3

QUESTIONS ON AREAS AND VOLUMES

EXERCISE 5A

1) A triangle has a base of length 35cm and a vertical height of 45cm, find its area.

2) A circle has a diameter of 10cm, find the area of the circle. (NB: diameter = radius × 2)

3) If the area of a circle is 25cm find the area of the minor sector which has an angle of 52 degrees.

4) A parallelogram has a base of 15 metres and a vertical height of 12 metres: find its area.

5) Calculate the volume of a cylinder which has a radius of 6cm and a length of 18cm.

6) If a pyramid has a square base of length 2cm and a height of 12 cm; find

a) The area of the cross section

b) The volume of the pyramid

7) A sphere has a radius of 8.5cm. Calculate the volume of the sphere.

EXERCISE 1A

1) $\frac{3}{25}$
2) $\frac{21}{26}$
3) $\frac{1}{40}$
4) $\frac{14}{33}$
5) $\frac{27}{46}$
6) $1\frac{46}{49}$
7) $\frac{1}{10}$

EXERCISE 1B

1) $\frac{15}{32}$
2) $\frac{11}{13}$
3) $19\frac{1}{6}$
4) $\frac{18}{343}$
5) $1\frac{19}{21}$
6) $\frac{45}{56}$
7) $\frac{1}{5}$

EXERCISE 1C

1) $\frac{17}{30}$
2) $1\frac{4}{15}$
3) $\frac{103}{112}$
4) $\frac{5}{7}$

EXERCISE 1D

1) $\frac{9}{80}$
2) $7\frac{21}{22}$
3) $\frac{37}{50}$
4) $\frac{2}{7}$

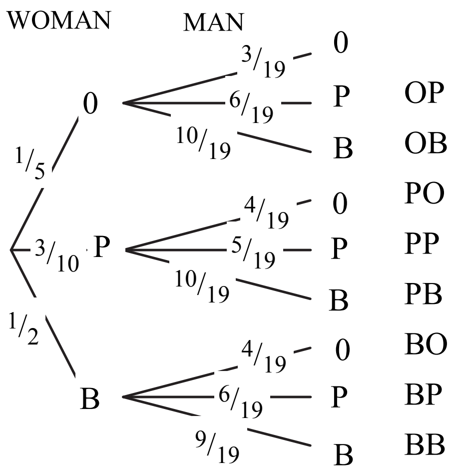
EXERCISE 1E

1) $\frac{5}{11}$
2) $\frac{22}{27}$

EXERCISE 2A

1) $\frac{1}{13}$
2) $\frac{1}{13}$
3) $\frac{1}{13} \cdot \frac{1}{17} = \frac{1}{221}$

EXERCISE 2B

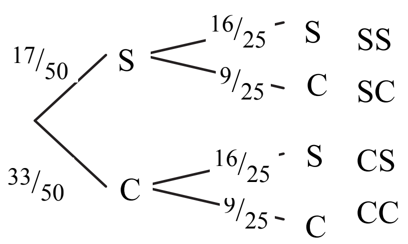


KEY: O = Orange
P = Pink
B = Brown

2) $\frac{1}{2} \cdot \frac{9}{19} = \frac{9}{38}$
3) $\frac{1}{5} \cdot \frac{6}{19} = \frac{6}{95}$
4) $(\frac{3}{10} \cdot \frac{5}{19}) + (\frac{3}{10} \cdot \frac{10}{19}) + (\frac{1}{2} \cdot \frac{6}{19}) + (\frac{1}{2} \cdot \frac{9}{19}) = \frac{12}{19}$
5) $1 - \frac{12}{19} = \frac{7}{19}$
6) $(\frac{1}{5} \cdot \frac{6}{19}) + (\frac{3}{10} \cdot \frac{4}{19}) + (\frac{3}{10} \cdot \frac{10}{19}) + (\frac{1}{2} \cdot \frac{6}{19}) = \frac{42}{95}$
7) $(\frac{3}{10} \cdot \frac{5}{19}) = \frac{3}{38}$
8) $\frac{42}{95} + \frac{3}{38} = \frac{99}{190}$

EXERCISE 2C

1) 17
2) 5 YRS. 12 YRS.



3) $\frac{297}{1250}$
4) $\frac{264}{625}$

EXERCISE 3A

1a) 1, 2, 3, 5, 6, 10, 15, 30
b) 1, 2, 7, 14
c) 1, 2, 4
d) 1, 5, 25
e) 1, 2, 3, 4, 6, 9, 12, 18, 36
f) 1, 3, 9
g) 1, 2, 3, 6, 9, 18
h) 1, 2, 5, 10
i) 1, 2, 5, 10, 25, 50

EXERCISE 3B

1a) 4, 8, 12, 16, 20
b) 5, 10, 15, 20, 25
c) 12, 24, 36, 48, 60
d) 10, 20, 30, 40, 50
e) 2, 4, 6, 8, 10
f) 9, 18, 27, 36, 45
2a) 6, 12, 18, 24, 30, 36, 42
b) 8, 16, 24, 32, 40, 48, 56
c) 14, 28, 42, 56, 70, 84, 98
d) 7, 14, 21, 28, 35, 42, 49
e) 13, 26, 39, 52, 65, 78, 91
f) 100, 200, 300, 400, 500, 600,700

EXERCISE 3C

1a) $14 = 2 \cdot 7$
b) $22 = 2 \cdot 11$
c) $36 = 2 \cdot 2 \cdot 3 \cdot 3$
d) $50 = 2 \cdot 5 \cdot 5$
e) $33 = 3 \cdot 11$
f) $42 = 2 \cdot 3 \cdot 7$
g) $45 = 3 \cdot 3 \cdot 5$
h) $60 = 2 \cdot 2 \cdot 3 \cdot 5$
i) $5 = 5$

EXERCISE 3D

1a) 42
b) 36
c) 44
d) 52
e) 30
f) 6
2a) 26
b) 11
c) 3
d) 12
e) 1
f) 4

EXERCISE 4A

1) 51.4°
2) 120°
3) 144°
4) 120°

EXERCISE 4B

1a) 32.7°
b) 147.3°
c) 12
d) 5

EXERCISE 4C

1a) 60°
b) 6, hexagon
2) 14 sides

EXERCISE 5A

1) 787.5 cm^2
2) 78.5 cm^2
3) $3\frac{11}{18}$ cm^2
4) 180 m^2
5) 2036 cm^3
6a) 4 b) 16
7) 2572 cm^3