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NOTE: All diagrams are deliberately approximate. This is to ensure that the answers cannot be easily calculated by simply measuring the lengths and angles from the diagrams, and also to test the students' ability to accurately draw diagrams from given information.

MATHEMATICS PACK 2

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GEOMETRY

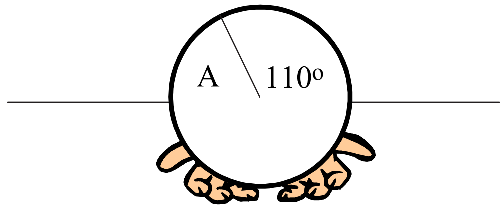
Geometry problems may involve finding angles which are related to a circle, a quadrilateral or are produced by lines crossing each other. Before attempting geometry problems you should always start by looking at the information you are given and relate this to the basic facts given below. If you do not know the following facts geometry problems become difficult to solve.

⌘ ANGLES ON A STRAIGHT LINE

The first fact is that the sum of the angles which are on a straight line is always 180^o.

WORKED EXAMPLE

Calculate the value of angle A shown in the diagram.



As the sum of the angles has to be 180^o you perform the following calculation.

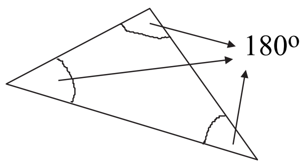
Angle A + $110^o = 180^o$

Angle A = $180^o - 110^o$

Angle A = 70^o

⌘ ANGLES IN A TRIANGLE

Whether the triangle is right-angled or not, the sum of the three angles in a triangle is always 180^o.



WORKED EXAMPLE

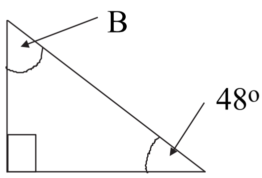
Find the value of angle B in the right angled triangle shown in the diagram.

Angle B + $48^o + 90^o = 180^o$

Therefore

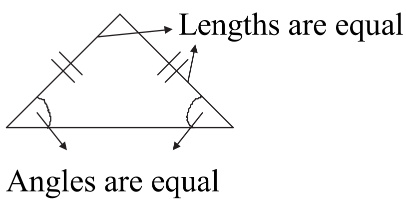
Angle B = $180^o - 90^o - 48^o$

Angle B = 42^o



⌘ ISOSCELES TRIANGLES

If you are told that a triangle is isosceles the triangle has two sides which are equal in length and two angles which are also equal.



WORKED EXAMPLE

If an isosceles triangle has an angle of 25° find the values of the two other angles, given they are equal.

The sum of all three angles has to be 180° so you perform the following calculations:

$180^\circ - 25^\circ$ = Two missing angles

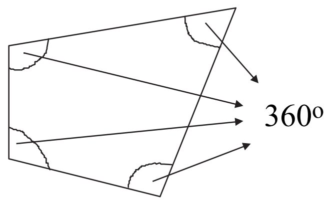
Sum of two missing angles = 155°

As the two angles are equal you divide by two to find the value of each angle.

$155^\circ \div 2 = 77.5^\circ$

⌘ANGLES IN A QUADRILATERAL

The sum of all the angles in a quadrilateral, a four sided shape, is always 360°.



WORKED EXAMPLE

A quadrilateral has angles of 36°, 55° and 105°. Find the value of the missing angle.

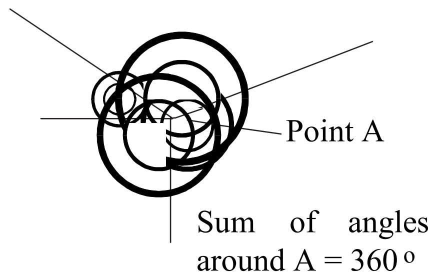
The sum of all four angles has to be 360° so you perform the following calculation to find the missing angle.

Missing angle + 36 + 55 + 105 = 360°

Missing angle = 360 - 36 - 55 - 105

Missing angle = 164°

⌘ANGLES AROUND A POINT



The sum of all the angles around a particular point is always 360°.

WORKED EXAMPLE

If a point P is surrounded by angles of value 45° and 203° what is the value of the missing angle X.

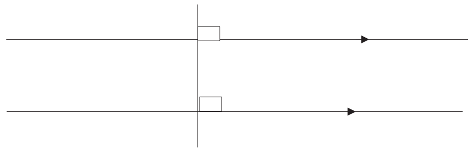
The sum of the angles around point P has to be 360°.

Therefore $45^\circ + 203^\circ$ + angle X = 360°

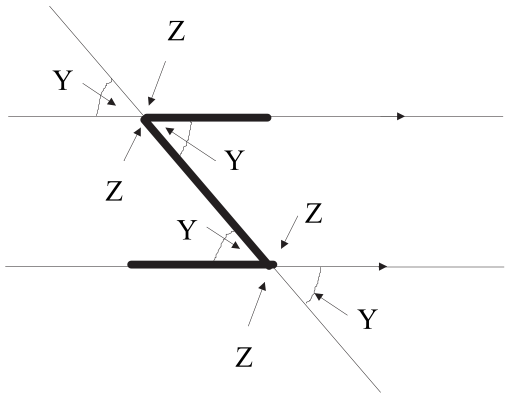
Angle X = $360^\circ - 203^\circ - 45^\circ$

Angle X = 112°

⌘ When a line crosses two other lines at right angles: 90° the two lines are parallel.

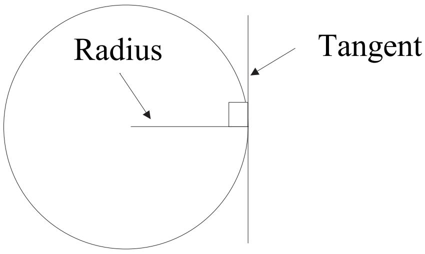


⌘ When parallel lines are crossed by another line the angles opposite each other are equal.

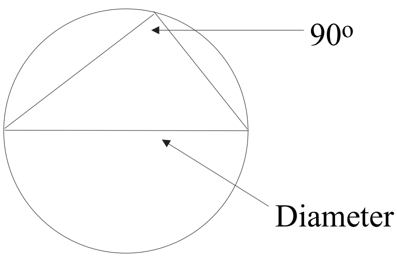


⌘ As shown in the above diagram angles in the Z are equal. The Z has been highlighted above.

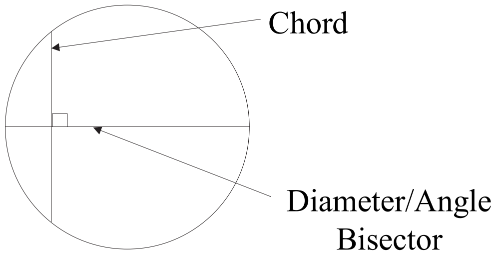
⌘ A line touching a circle, at only one point, on the circumference of the circle is called a tangent. Tangents are always at right angles to the radius of the circle.



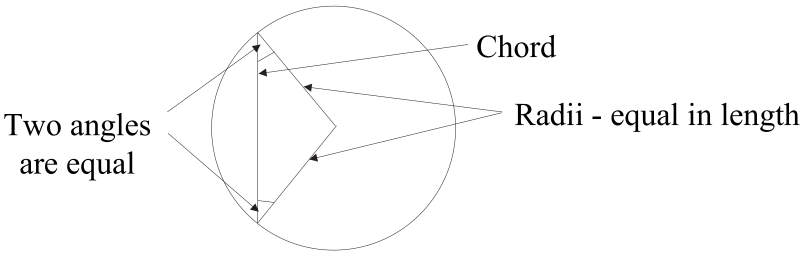
⌘ If a triangle is drawn inside a circle so that the base of the triangle is equal to the diameter of the circle, the angle in the triangle touching the circumference of the circle is always 90°



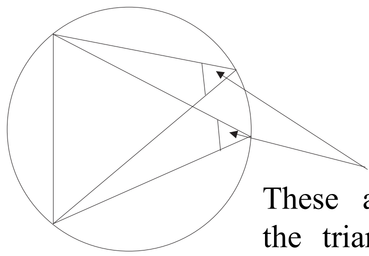
⌘If a line is drawn across a circle at any point in the circle it is called a chord. If a line cuts the chord at 90° the line is an angle bisector and must be the diameter of the circle.



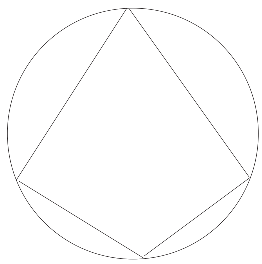
⌘When two radii are drawn in a circle and are joined to a chord the triangle formed is isosceles. This is because the two radii are of equal lengths and two angles in the triangle are also equal.



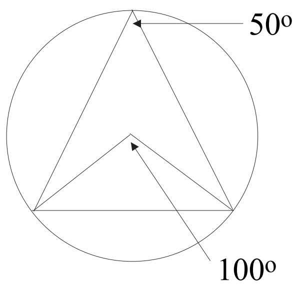
⌘A chord divides a circle into two segments. If two triangles are drawn in the same segment, so that they both have the chord as their base length, the angles in the triangles which touch the circumference of the circle are equal in value.



⌘CYCLIC QUADRILATERALS - these are quadrilaterals which have been drawn inside a circle so that the corners of the quadrilateral touch the circumference of the circle at one point. The angles opposite each other in a cyclic quadrilateral always add up to 180°.

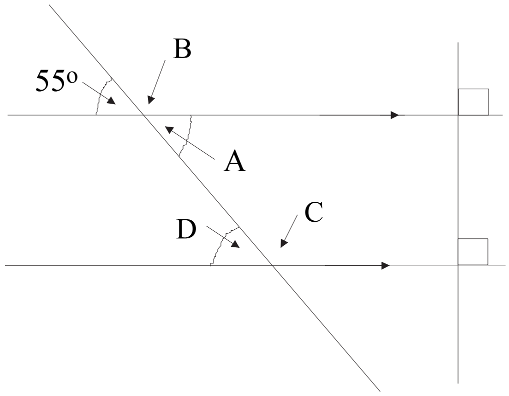


⌘If two triangles are drawn inside a circle so that they both have a chord as their base length; so that one triangle forms an angle at the centre of the circle and the other triangle forms an angle at a point on the circumference of the circle, the angle at the centre of the circle is always double the value of the angle at the circumference. The diagram illustrates this with an example of the angle at the centre being 100°, which is double the angle at the circumference.



WORKED EXAMPLES ON GEOMETRY 5

1) Find the value of angles A, B, C and D.



Before attempting to answer this question you should start by looking at the information given. By looking at the above diagram the first thing you should notice is that there are two right angles formed from a line crossing two other lines; the two lines must therefore be parallel. To help you remember that the two lines are parallel you should add arrows to your diagram.

One of the facts mentioned earlier states:

When parallel lines are crossed by another line the angles opposite each other are equal.

We can now apply this fact to find the value of angle A. Angle A is opposite an angle of 55 ^{o} so angle A must also have a value of 55 ^{o}.

Angle A = 55 ^{o}

You should then find the value of angle B. Angle B is on a straight line so you should use the fact that the sum of the angles which are on a straight line is always 180 ^{o}. Therefore:

Angle B + 55 ^{o} = 180^o

So Angle B = 180 ^{o} - 55^o

Angle B = 125 ^{o}

To find the value of angle D you should use the fact that angles in the Z are equal. It is easy to see that angle D must be equal to angle A as they are both in the Z.

Angle D = Angle A

Angle D = 55 ^{o}

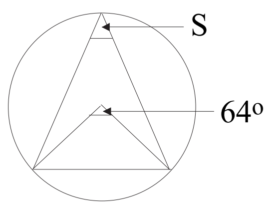
Angle C is part of a straight line with angle D so is found as follows:

Angle C + Angle D = 180 ^{o}

Angle C = 180 ^{o} - Angle D

Angle C = 125 ^{o}

2) Find the value of angle S shown in the diagram.



To answer this question you should use the fact that the angle at the centre of the circle is always double the value of the angle at the circumference. Therefore angle S must be half the value of the angle at the centre.

Angle S = 64 ^{o} ÷ 2

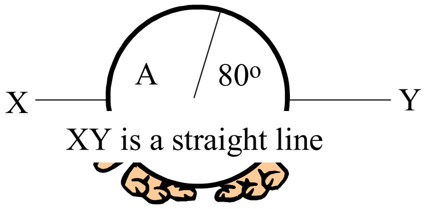
Angle S = 32 ^{o}

QUESTIONS ON GEOMETRY 6

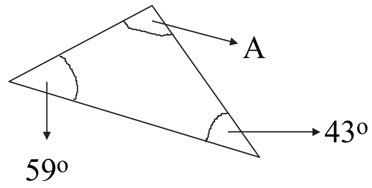
EXERCISE 1

1) Calculate the value of angle A in each of the following diagrams.

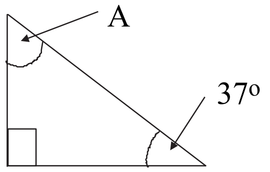
(a)



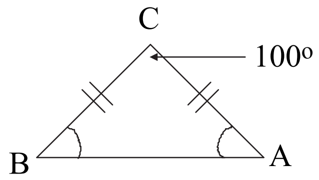
(b)



(c)

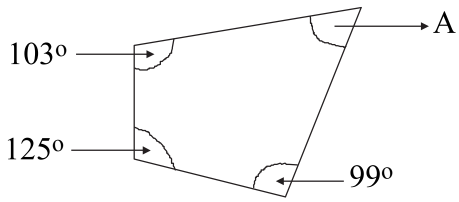


(d)

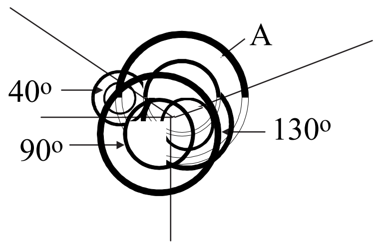


Triangle ABC is isosceles

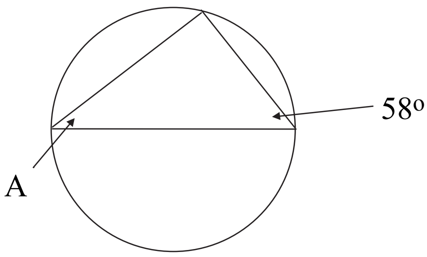
(e)



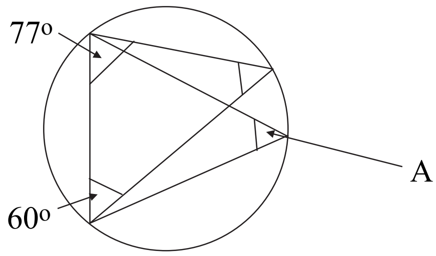
(f)



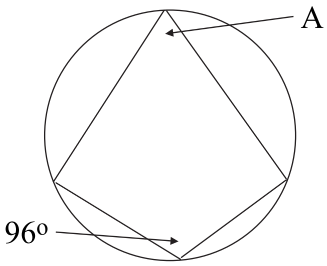
(g)



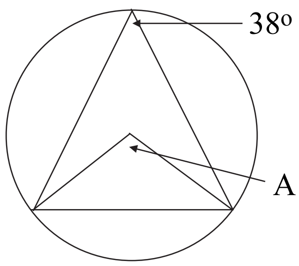
(h)



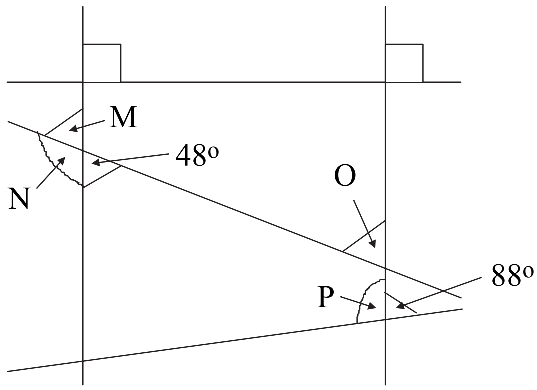
(i)



(j)



2) Find the value of angle M, N, O, and P in the diagram below.



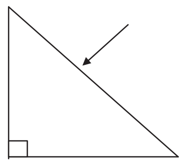
PYTHAGORAS' THEOREM 7

Pythagoras' theorem is used to answer questions in which the missing length of a right-angled triangle needs to be found.

Pythagoras' theorem states that the square of the longest side in a triangle is equal to the sum of the squares of the two shorter sides.

i.e. Longest side^2 = Shorter side^2 + Shorter side^2

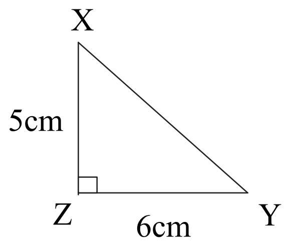
To apply the above equation to find a missing length you first need to know that the longest side in a right-angled triangle is ALWAYS facing the right-angle.



Below are some worked examples which show you how to apply Pythagoras' theorem.

WORKED EXAMPLE 1

Find the length of XY in the triangle shown.



The longest side in this triangle is clearly XY as it is facing the right-angle. The two shorter sides must therefore be XZ and YZ.

Longest side^2 = Shorter side^2 + Shorter side^2

Therefore: $XY^2 = XZ^2 + YZ^2$

Therefore: $XY^2 = 5^2 + 6^2$

$XY^2 = 25 + 36 = 61$

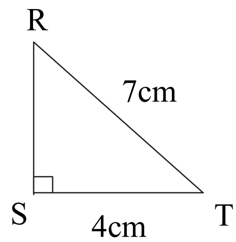
$XY = \sqrt{61}$

XY = 7.81cm (to three significant figures)

WORKED EXAMPLE 2

Find the length of RS in the triangle shown, giving your answer to 3 significant figures.

The longest length in this triangle is clearly RT as it is facing the right-angle.
As RS is not the longest length we first rearrange the following equation:



Longest side^2 = Shorter side^2 + Shorter side^2

$RT^2 = ST^2 + RS^2$

It should read:

$RS^2 = RT^2 - ST^2$

$RS^2 = 7^2 - 4^2$

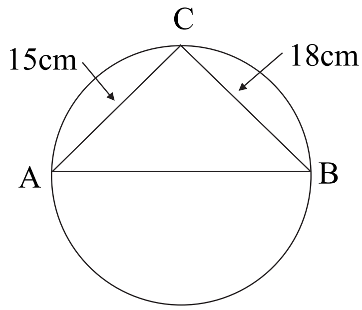
$RS^2 = 49 - 16 = 33$

$RS = \sqrt{33}$

RS = 5.74cm (to three significant figures).

WORKED EXAMPLE 3

Pythagoras' theorem can also be applied to geometry questions such as the one below.



Find length AB, the diameter of the circle, to three significant figures and give the units.

To answer this question you should first recognise that the triangle drawn in the circle is right-angled. As if a triangle is drawn inside a circle so that the base of the triangle is equal to the diameter of the circle, the angle in the triangle touching the circumference of the circle is always 90^{o}

The longest side in this triangle is therefore AB so the calculation performed is as follows.

$AB^2 = AC^2 + BC^2$

$AB^2 = 15^2 + 18^2$

$AB^2 = 225 + 324 = 549$

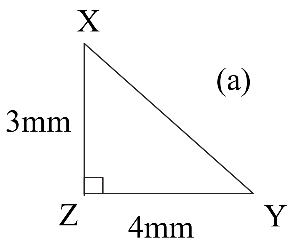
$AB = \sqrt{549}$

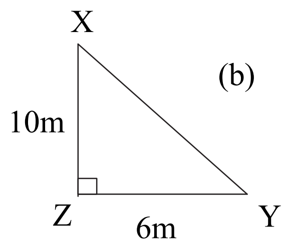
AB = 23.4cm (to three significant figures).

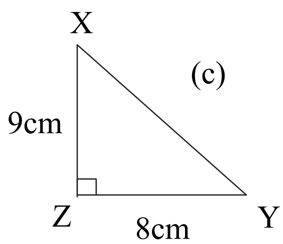
QUESTIONS ON PYTHAGORAS' THEOREM

EXERCISE 2

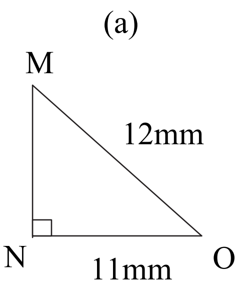
1) Find the length of XY in each of the following triangles using Pythagoras' theorem, giving your answers to three significant figures.

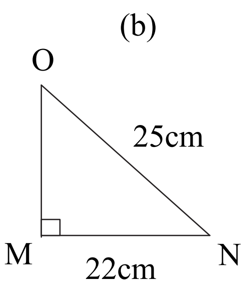


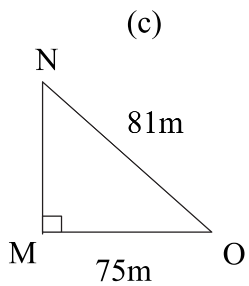




2) Find the length of MN in each of the following triangles using Pythagoras' theorem, giving your answers to three significant figures.







TRIGONOMETRY

It is possible to find a missing length in a right-angled triangle if you know the lengths of the two other sides, using Pythagoras' theorem. However, Pythagoras' theorem does not allow you to find the length of a particular side, in a right angled triangle, given only one length and the appropriate angle but trigonometry does.

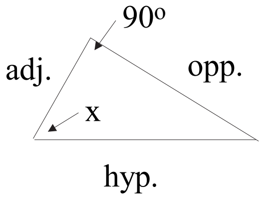
Trigonometry questions involve finding either the value of an angle in a right-angled triangle given two appropriate lengths, or the length of a side being given the value of the appropriate angle and a length.

It is important that you know the sides of a triangle can be referred to as hypotenuse, opposite and adjacent.

The HYPOTENUSE is always the side opposite the right angle.

OPPOSITE is always the side facing the angle being used in the equation.

ADJACENT is the side next to the angle being used in the triangle.



Trigonometry questions are quite simple to solve if you learn the following phrase as it helps you to learn the equations necessary to solve the questions. The phrase is:

SOHCAHTOA

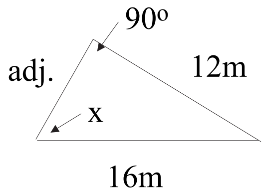
'SOH' should help you remember that Sin 'x' = Opposite ÷ Hypotenuse

'CAH' should help you remember that Cos 'x' = Adjacent ÷ Hypotenuse

'TOA' should help you remember that Tan 'x' = Opposite ÷ Adjacent

⌘ You would use SOH to help you answer the following questions.

1) Find the value of angle X in the triangle shown.



From the diagram you can see that we have been told the length of the hypotenuse and the length of the side opposite angle 'x.'

Hypotenuse (H) = 16m

Opposite (O) = 12m

You therefore look at the phrase SOHCAHTOA to see which part of the phrase includes both the hypotenuse and opposite. It is clearly SOH. You then write down what SOH stands for:

Sin 'x' = Opposite ÷ Hypotenuse

You should now substitute the values given into the above equation:

Sin 'x' = 12 ÷ 16

Sin 'x' = 0.75

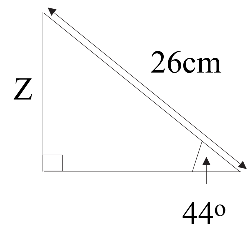
You can now find the value of angle 'x' on your calculator by finding the value of:

inverse sin 0.75 = 48.6 °

Angle X = 48.6 ° (to one decimal place)

2) Find the length of Z in the triangle below.

From the diagram you can see that we have been given the length of the hypotenuse and the value of the angle opposite Z.



Hypotenuse (H) = 26cm

Angle opposite Z = 44 ^{o}

You therefore look at the phrase SOHCAHTOA to see which part of the phrase includes both the hypotenuse and opposite. It is clearly SOH. You then write down what SOH stands for:

Sin Z = Opposite ÷ Hypotenuse

You should now substitute the values given into the above equation:

Sin 44 = Z ÷ 26

To find the length of Z you have to rearrange the above equation so that it reads:

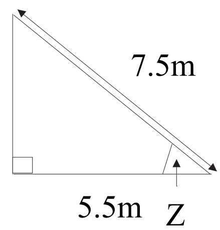
Z = Sin 44 x 26

Z = 18.1cm (to one decimal place).

⌘ Below is an example of a question in which you would use CAH to help you solve it.

1) Find the value of angle Z in the triangle below giving your answer to one decimal place.

By looking at the diagram you can see that we are given the length of the hypotenuse and the length of the side adjacent to the angle Z.



Hypotenuse = 7.5m Adjacent = 5.5m

You therefore look at the phrase SOHCAHTOA to see which part of the phrase includes both the hypotenuse and adjacent. It is clearly CAH. You then write down what CAH stands for:

Cos Z = Adjacent ÷ Hypotenuse

You should now substitute the values given into the above equation:

Cos Z = 5.5 ÷ 7.5

Cos Z = 0.7333

You can now find the value of angle Z on your calculator by finding the value of:

inverse cos 0.7333 = 48.6 ^{o}

Angle X = 42.8 ^{o} (to one decimal place)

⌘ Questions using TOA can be done using the same methods as described above but this time if you want to find an angle you substitute into the equation:

Tan 'x' = Opposite ÷ Adjacent

Once you find the value of Tan 'x' you can find the value of angle 'x' on your calculator by doing inverse tan... If you want to find the a length using TOA you rearrange the equation.

e.g. to find the length of the side opposite the angle given the equation reads:

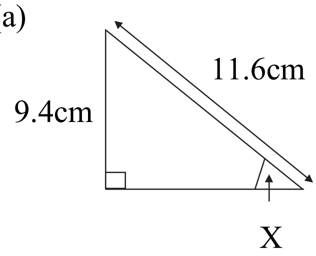
Opposite = Tan 'x' x Adjacent

QUESTIONS ON TRIGONOMETRY

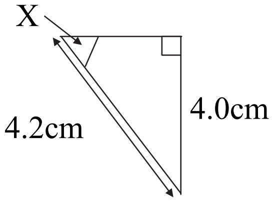
EXERCISE 3

1) Find the value of angle X in the following diagrams using the sin rule, giving your answer to one decimal place.

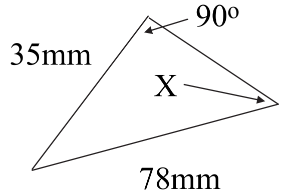
(a)



(b)

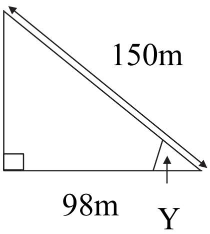


(c)

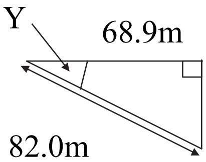


2) Find the value of angle Y in each of the following to one decimal place using the cos rule.

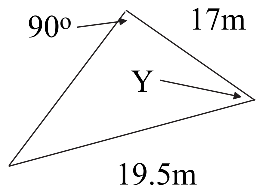
(a)



(b)

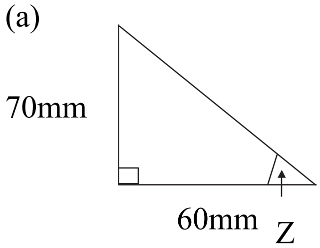


(c)

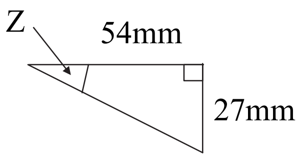


3) Find the value of angle Z in each of the following triangles to one decimal place using the tan rule.

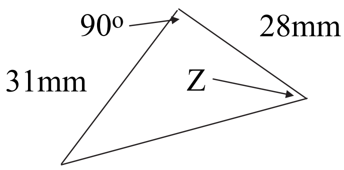
(a)



(b)

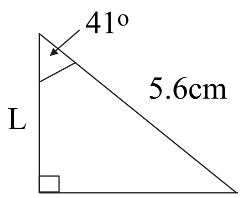


(c)

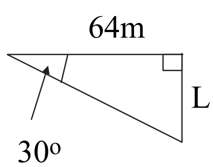


4) Find length L in each of the following triangles to three significant figures, giving units.

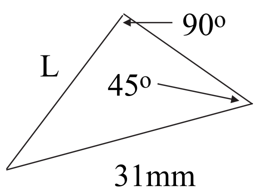
(a)



(b)



(c)

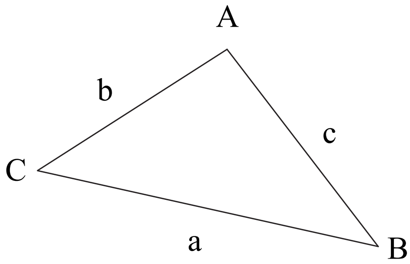


APPLICATION OF THE SINE RULE 12

So far we have seen that you can work out the value of an unknown angle or length in a right-angled triangle using trigonometry. However, the methods discussed earlier do not allow you to find the value of an unknown angle or length in a triangle which is not right-angled.

The Sine rule can be applied to all triangles including those which are not right-angled to find the value of an unknown angle or length; as long as another angle and its corresponding length in the triangle are given.

The triangle below shows which angle corresponds to which side of the triangle. The angle is always given in UPPER CASE *and the corresponding side is always given in* lower case.

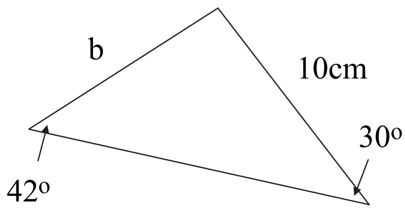


The Sine rule:

$$a \div \text{Sin } A = b \div \text{Sin } B$$

WORKED EXAMPLE 1

Using the information given in the triangle below calculate the length of 'b' to three significant figures.



To calculate the length of side 'b' you should first substitute the information given in the diagram into the equation.

$a \div \text{Sin } A = b \div \text{Sin } B$

$10 \div \text{Sin } 42 = b \div \text{Sin } 30$

You should then rearrange the equation so it reads:

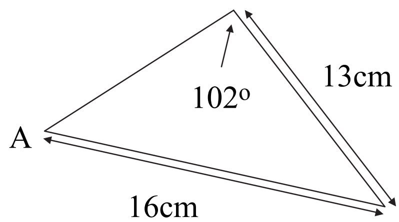
$b = (10 \div \text{Sin } 42) \times \text{Sin } 30$

You can then find the length of 'b' by performing the calculation on your calculator.

b = 7.47cm (to three significant figures)

WORKED EXAMPLE 2

Using the information given in the triangle below calculate the value of angle A to one decimal place.



When asked to find an angle you should first rearrange the equation so that it reads:

$\text{Sin } A \div a = \text{Sin } B \div b$

Then substitute the information given into the equation.

$\text{Sin } A \div 13 = \text{Sin } 102 \div 16$

$\text{Sin } A = (\text{Sin } 102 \div 16) \times 13$

$\text{Sin } A = 0.7947$

$A = 52.6^{\circ}$ (to one decimal place)

APPLICATION OF THE COSINE RULE

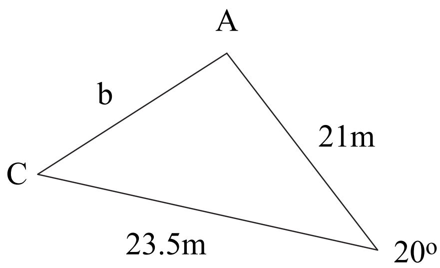
The Cosine rule like the Sine rule can be applied to all triangles to find the length of an unknown side given its corresponding angle and the lengths of the other two sides. The Cosine rule can also be used to find the value of an unknown angle in the triangle if you are given the lengths of all three sides.

The Cosine rule:

$$a^2 = b^2 + c^2 - (2 \cdot b \cdot c \cdot \text{Cos } A)$$

In the above equation 'a' is the length of the unknown side, 'b' and 'c' are the other two sides and their lengths are given in the question. 'A' is the corresponding angle to the unknown side 'a'.

<u>WORKED EXAMPLE 1</u>



Using the information in the diagram calculate the length of side 'b'.

You first need to rearrange the equation so that it reads:

$b^2 = a^2 + c^2 - (2 \times a \times c \times \text{Cos } B)$

You then substitute the values given in the diagram into the equation.

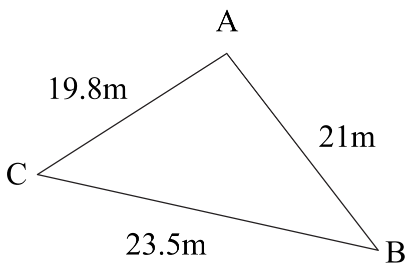
$b^2 = 23.5^2 + 21^2 - (2 \times 23.5 \times 21 \times \text{Cos } 20)$

$b^2 = 993.25 - 927.48$

$b^2 = 65.77$

$b = 8.11\text{m}$ (to the three significant figures)

<u>WORKED EXAMPLE 2</u>



Calculate angle A in the triangle below using the information in the diagram.

You first need to rearrange the equation for the Cosine rule as follows:

$a^2 = b^2 + c^2 - (2 \times b \times c \times \text{Cos } A)$

$(2 \times b \times c \times \text{Cos } A) + a^2 = b^2 + c^2$

$(2 \times b \times c \times \text{Cos } A) = b^2 + c^2 - a^2$

$\text{Cos } A = (b^2 + c^2 - a^2) \mid (2 \times b \times c)$

Now substitute in the values given and calculate the value of angle A.

$\text{Cos } A = (19.8^2 + 21^2 - 23.5^2) \mid (2 \times 19.8 \times 21)$

$\text{Cos } A = 280.79 \div 831.6$

$\text{Cos } A = 0.3377$

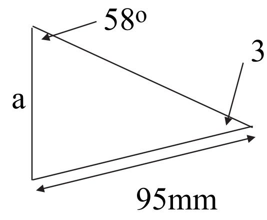
$A = 70.3^{\circ}$ (to one decimal place)

QUESTIONS ON THE SINE RULE AND COSINE RULE

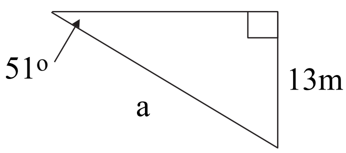
EXERCISE 4

1) Find the length of side 'a' in each of the following triangles using the Sine rule. Give your answer to three significant figures.

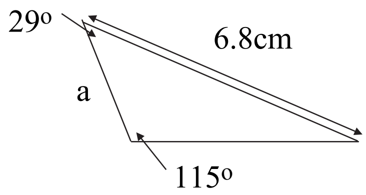
(a)



(b)

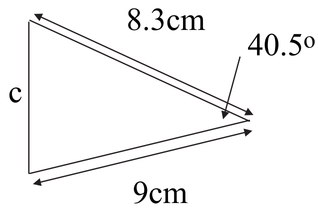


(c)

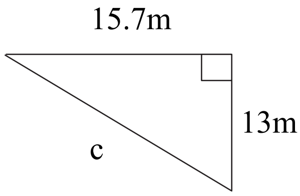


2) Find the length of side 'c' in each of the following triangles using the Cosine rule. Give your answer to three significant figures.

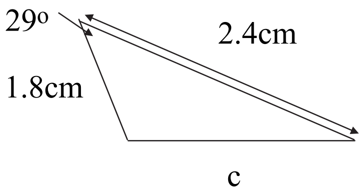
(a)



(b)

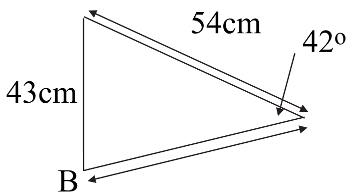


(c)

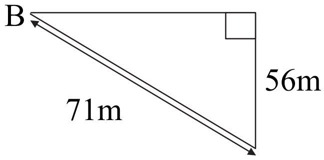


3) Find the value of angle B in the following triangles. Give you answer to one decimal place.

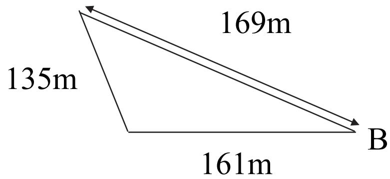
(a)



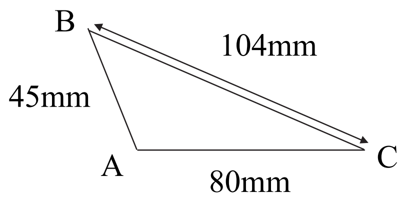
(b)



(c)

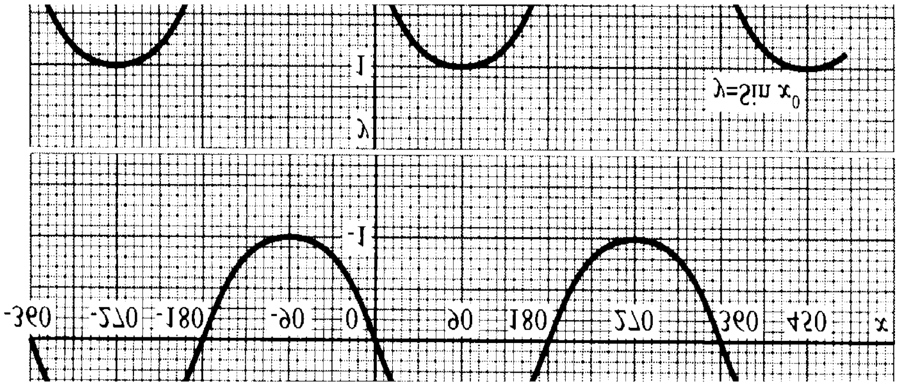


4) Find the value of angles A, B and C in the triangle shown. Give your answer to one decimal place.

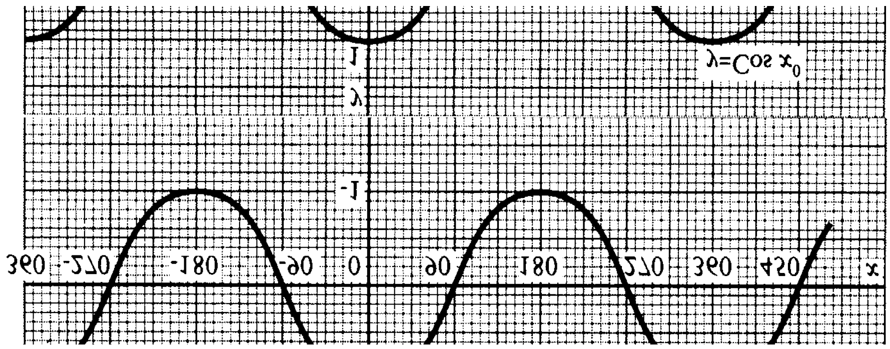


SIN, COS AND TAN GRAPHS

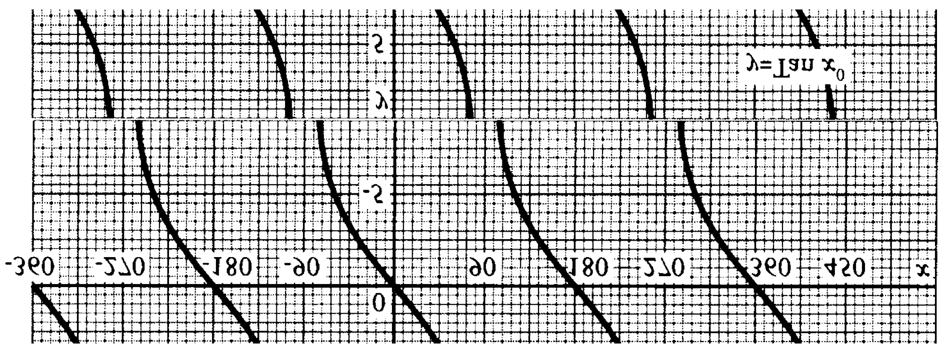
Below is a graph of y = sin x. You should notice that the maximum is always 1 and the minimum is always -1. The graph repeats itself every 360 °.



Below is a graph of y = cos x. It is similar to the graph above as this graph also has a maximum value of 1 and a minimum value of -1. It repeats every 360 °.



Below is a graph of y = tan x. The graph goes to positive infinity and negative infinity on either side of the asymptotes. Unlike the others this graph repeats every 180 °.



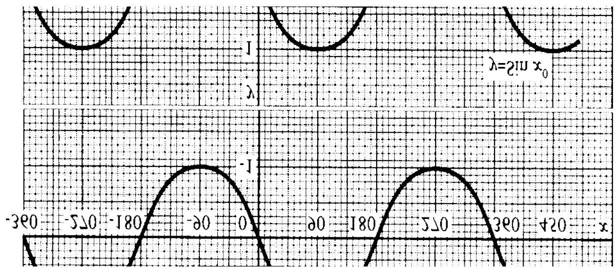
An example of a question which you may be faced with on the Sin graph is shown below with points on how to go about tackling it.

WORKED EXAMPLE 1

For the graph of Y = Sin X where X ranges from 0 to 360 find all the values of X when

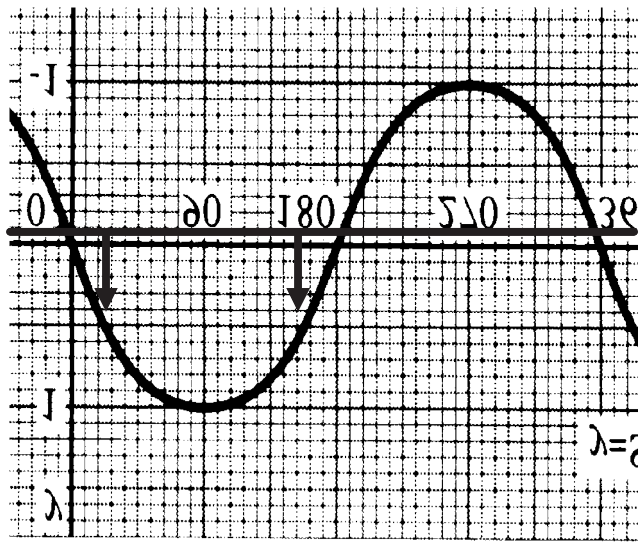
Sin X = 0.5.

⌘ First sketch the y = sin x graph for the range given in the question: from x = 0 ^{o} to x = 360^{o}.



⌘ Use a ruler to draw the horizontal line

y = 0.5 across the graph, as shown in the diagram.



⌘ You will notice that the horizontal line crosses the graph of y = sin x in two places. Vertical lines should be drawn from these two points of intercept down until they cross the x-axis.

⌘ The values on the x-axis where the vertical lines cross it are the solutions to Sin X = 0.5. To find what these values are you should first use your calculator and do inverse sin of 0.5. That will give you an answer of 30 ^{o}. This is one solution.

⌘ As your calculator does not give you the other solution you now need to look at the symmetry of the graph. It can be seen that the distance between the line crossing the x-axis and 180^{o} is also 30^{o}. You can therefore find the second solution by performing the following calculation:

180^{o} - 30^{o} = 150^{o}

⌘ To check that this second solution is correct you can put sin 150 into your calculator and your answer should be 0.5.

⌘ Therefore Sin X = 0.5 when X = 30^{o} and when X = 150^{o} where X ranges from 0 to 180 degrees.

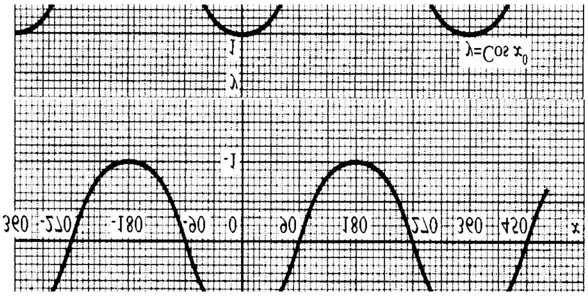
An example of a question which you may be faced with on the Cos graph is shown below with points on how to go about tackling it.

<u>WORKED EXAMPLE 2</u>

For the graph of Y = Cos X where X ranges from -180 to 180 find all the values of X when

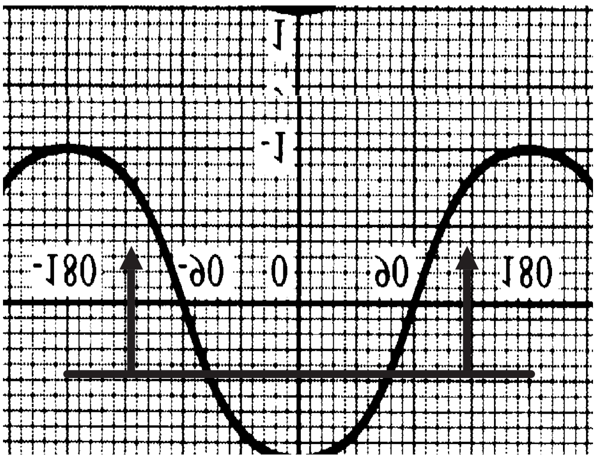
Cos X = -0.8.

⌘ First sketch the y = cos x graph for the range given in the question: from x = -180 to x = 180º.



⌘ Use a ruler to draw the horizontal line

y = -0.8 across the graph, as shown in the diagram.



⌘ You will notice that the horizontal line crosses the graph of y = cos x in two places. Vertical lines should be drawn from these two points of intercept up until they cross the x-axis.

⌘ The values on the x-axis where the vertical lines cross it are the solutions to Cos X = -0.8. To find what these values are you should first use your calculator and do inverse cos of -0.8. That will give you an answer of 143.1 º. This is one solution.

⌘ As your calculator does not give you the other solution you now need to look at the symmetry of the graph. It can be seen that the second solution is exactly the same distance from the y-axis as the first solution but as it is in the negative region of the graph the solution must be -143.1º.

⌘ To check that this second solution is correct you can put cos -143.1 into your calculator and your answer should be -0.8.

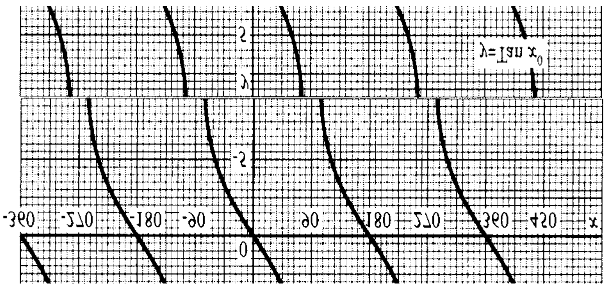
⌘ Therefore Cos X = -0.8 when X = 143.1 º and when X = -143.1 º where X ranges from -180 to 180 degrees.

An example of a question which you may be faced with on the Tan graph is shown below with points on how to go about tackling it.

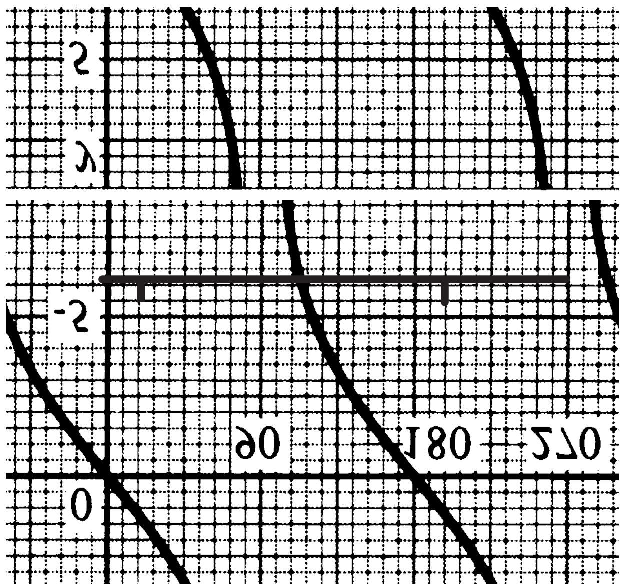
<u>WORKED EXAMPLE 1</u>

For the graph of Y = Tan X where X ranges from 0 to 270 find all the values of X when

Tan X = 1.



⌘ First sketch the y = tan x graph for the range given in the question: from x = 0 ^{o} to x = 270^{o}.



⌘ Use a ruler to draw the horizontal line

y = 1 across the graph, as shown in the diagram.

⌘ You will notice that the horizontal line crosses the graph of y = tan x in two places. Vertical lines should be drawn from these two points of intercept down until they cross the x-axis.

⌘ The values on the x-axis where the vertical lines cross it are the solutions to Tan X = 1. To find what these values are you should first use your calculator and do inverse tan of 1. That will give you an answer of 45 ^{o}. This is one solution.

⌘ As your calculator does not give you the other solution you now need to look at the symmetry of the graph. It can be seen that the distance between the line crossing the x-axis and 180^{o} is also 45 ^{o}. You can therefore find the second solution by performing the following calculation:

180^{o} + 45^{o} = 225^{o}

⌘ To check that this second solution is correct you can put tan 225 into your calculator and your answer should be 1.

⌘ Therefore Tan X = 1 when X = 45 ^{o} and when X = 225 ^{o} where X ranges from 0 to 270 degrees.

QUESTIONS ON THE SIN, COS & TAN GRAPH

EXERCISE 5

1) Draw the following graphs where X ranges from -360^{o} to 360^{o}.

a) Y = Sin X b) Y = Cos X c) Y = Tan X

2) For the graph of y = cos x find all the values of x when cos x = -1, where x ranges from -360^{o} to 360^{o}.

3) For the graph of y = sin x find all the values of x when sin x = 1, where x ranges from -360^{o} to 360^{o}.

4) For the graph of y = tan x find all the values of x when tan x = -1, where x ranges from -360^{o} to 360^{o}.

5) In the following questions X ranges from -180^{o} to 360^{o}. Give all your answers to one decimal place.

a) Solve sin x = -0.75

b) Solve sin x = 0.2

c) Solve sin x = $-\,^{1}/_{3}$

d) Solve cos x = 0.5

e) Solve cos x = 0.75

f) Solve cos x = $-\,^{4}/_{5}$

g) Solve tan x = $-\,^{1}/_{4}$

h) Solve tan x = -0.5

i) Solve tan x = 0.6

6) In the following questions X ranges from -180^{o} to 270^{o}. Give all your answers to one decimal place.

a) Solve sin x = 0.4

b) Solve cos x = -0.9

c) Solve tan x = 1.5

d) Solve cos x = 0.45

e) Solve tan x = $^{1}/_{3}$

f) Solve sin x = $-\,^{3}/_{5}$

EXERCISE 1

1a) 100^o

b) 78^o

c) 53^o

d) 40^o

e) 33^o

f) 100^o

g) 32^o

h) 43^o

i) 84^o

j) 76^o

2) M = 48^o

N = 132^o

O = 48^o

P = 92^o

EXERCISE 2

1a) 5mm

b) 11.7m

c) 12.0cm

2a) 4.80mm

b) 11.9cm

c) 30.6m

EXERCISE 3

1a) 54.1^o

b) 72.2^o

c) 26.7^o

2a) 49.2^o

b) 32.8^o

c) 29.3^o

3a) 49.4^o

b) 26.6^o

c)47.9^o

4a) L = 4.23 cm

b) L = 37.0 m

c) L = 21.9 m

EXERCISE 4

1a) a = 67.4mm

b) a = 16.7 m

c) a = 4.41cm

2a) c = 6.02 cm

b) c = 20.4 m

c) c = 1.44 cm

3a) B = 57.2^o

b) B = 52.1^o

c) A = 70.6^o

4) B = 46.5^o

C = 22.8^o

A = 110.7^o

EXERCISE 5

2) X = 180^o, X = -180^o

3) X = 90^o, X = -270^o

4) X = -45^o, X = -225^o, X = 135^o, X = 315^o

5a) X = -48.6^o, X = -131.4^o, X = 228.6^o, X = 311.4^o

b) X =11.5^o, X = 168.5^o

c) X = -19.5^o, X = -160.5^o, X = 199.5^o, X = 340.5^o

d) X =60^o, X = -60^o, X = 300^o

e) X = 41.4^o, X = -41.4^o, X = 318.6^o

f) X = 143.1^o, X = -143.1^o, X = 216.9^o

g) X = -14.0^o, X = 166.0^o, X = 346.0^o

h) X = -26.6^o, X = 153.4^o, X = 333.4^o

i) X = 31.0^o, X = 211.0^o, X = -149.0^o

6a) X = 23.6^o, X = 156.4^o

b) X = 154.2^o, X = 205.8^o, X = -154.2^o

c) X = 56.3^o, X = -123.7^o, X = 236.3^o

d) X =63.3^o,

X = -63.3^o

e) X = 18.4^o,

X = 198.4^o,

X = -161.6^o

f) X = -36.9^o,

X = -143.1^o,

X = 216.9^o